Inventor Mobility, Knowledge Diffusion, and Growth

Yasutaka Koike-Mori	Toshitaka Maruyama	Koki Okumura
UNC Chapel Hill	Bank of Japan	UCLA *

September 22, 2025

Abstract

We analyze the impact of inventor mobility between firms on knowledge diffusion, economic growth, and welfare. Using German patent data matched with employer-employee data, we provide evidence showing that inventor mobility between firms is associated with knowledge diffusion. Motivated by this evidence, we develop an endogenous economic growth theory in which inventors contribute to knowledge diffusion by moving between firms, in addition to engaging in internal R&D within firms. Using a model calibrated with German data, we first analyze the impact of non-compete clauses. We find that banning non-compete clauses reduces welfare by 0.38%. Furthermore, we show that optimal regulation of non-compete clauses leads to an allocation that closely approximates the social optimum and to a level of social welfare that nearly attains it. Finally, to analyze the impact of the decline in inventor mobility on economic growth over the past few decades, we calibrate the model to match the observed transition path of inventor mobility. Our results show that the decline in inventor mobility reduced growth from internal R&D by 0.04 pp and growth from knowledge diffusion by 0.20 pp, resulting in a total decrease in economic growth of 0.24 pp.

*We thank Andy Atkeson, David Baqaee, Serguey Braguinsky, Ariel Burstein, Levi Crews, Pablo Fajgelbaum, John Grigsby, Joao Guerreiro, Ekaterina Gurkova, Gary Hansen, Ryo Horii, Hugo Hopenhayn, Oleg Itskhoki, Hiroyuki Kubota, Theodore Naff, Koki Oikawa, Lee Ohanian, Liyan Shi, Takayuki Tsuruga, Jon Vogel, Conor Walsh, Pierre-Olivier Weill, and seminar participants at UCLA, SWET 2023, and the 2023 Moriguchi Prize for their valuable comments. We also thank the Institute for Employment Research for generously providing data. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Bank of Japan. This paper was awarded the 2023 Moriguchi Prize by the Institute of Social and Economic Research at Osaka University.

1 Introduction

Inventors play a crucial role in the diffusion of knowledge between firms as well as internal R&D activities within firms. As Arrow (1962) noted, the "mobility of personnel among firms provides a way of spreading information," highlighting that the movement of inventors across firms has long been considered a key factor in inter-firm knowledge diffusion[1]. However, the effects of policies that impact inventor mobility, such as regulations on non-compete clauses, on firm and economic growth have not been analyzed. This paper seeks to address the following questions: What framework is appropriate for analyzing the impact of inter-firm inventor mobility and the associated knowledge diffusion using microdata? How important is knowledge diffusion through inventor mobility for economic growth? What inefficiencies exist in the economy, and what policies should be implemented to address them?

This paper aims to make empirical, theoretical, and quantitative contributions to understanding the impact of inventor mobility between firms and the associated knowledge diffusion on firm and economic growth. Empirically, we utilize a novel dataset that links German matched employer-employee data with patent data for identified inventors. This dataset allows for the analysis of the characteristics of inventor mobility, the changes in wages associated with inventor movement, and the impact of inventor mobility on knowledge diffusion. Theoretically, we develop a new endogenous growth model that generates endogenous inventor job flows and knowledge diffusion networks. Then, we characterize the competitive equilibrium and optimal allocation, discuss the inefficiencies in this economy, and explore policies to address them. Quantitatively, we calibrate the model parameters based on our empirical findings and assess the impact of knowledge diffusion through inventor mobility on economic growth, the discrepancies between competitive equilibrium and optimal allocation, and the effects of regulations on non-compete clauses on welfare.

We begin by presenting empirical evidence on the job flows of inventors, suggesting the existence of knowledge diffusion. Specifically, we examine the mobility patterns of inventors—defined as workers who have created patents—using inventor biography data from Germany. These data link labor market biographies and their employing establishments, as recorded in German social security data, to patent register data. We find that a significant proportion of inventors move to less productive establishments. This result is robust across different productivity measures: establishment size, average wage,

[1]For evidence from recent studies, see Jaffe et al. (1993); Almeida and Kogut (1999); Song et al. (2003); Hoisl (2007); Rosenkopf and Almeida (2003); Breschi and Lissoni (2009); Singh and Agrawal (2011); Kaiser et al. (2015); Rahko (2017); Braunerhjelm et al. (2020).

and the number of patent citations. This job flow pattern for inventors contrasts with that of general workers, who are more likely to move to more productive firms, as established in previous literature (e.g., Haltiwanger et al. (2018)). In fact, using German matched employer-employee data, we show that inventors are significantly more likely to move from higher productivity firms to lower productivity firms compared to general workers. This result aligns with the idea that, unlike general workers, inventors tend to move between firms regardless of their marginal productivity, as their mobility facilitates knowledge diffusion. Furthermore, we find that inventors experience greater wage growth than general workers when changing jobs. This suggests that firms compensate for knowledge diffusion when hiring new inventors.

Our theoretical framework extends an endogenous growth model by incorporating a new element where knowledge diffuses as inventors engage in on-the-job search and move between firms. In this model, heterogeneous firms post vacancies while taking into account the knowledge spillovers from an inventor's previous employer. Inventors and firms are randomly matched in a frictional labor market. After a match is formed, inventors move if the match surplus is positive. This match surplus is then proportionally distributed between the new employer and the coalition of the inventor and previous employer. A key advantage of our model is that it endogenizes the inter-firm network of inventor mobility and knowledge spillovers, making it responsive to changes in economic conditions and policy interventions.

Similar to Klette and Kortum (2004), we assume that firms operate across multiple product lines and treat product lines and knowledge as equivalent. In their framework, Klette and Kortum (2004) equate the number of a firm's product lines with its "knowledge capital," and we adopt this assumption in our analysis: Innovation resulting from internal R&D activities is an increasing function of both the number of a firm's product lines and the employment of inventors. Through knowledge diffusion between firms and internal R&D activities, firms can accumulate more knowledge and expand their product lines.

One of our theoretical contributions is to make the model tractable by assuming that the number of product lines a firm produces takes continuous values and follows a specific stochastic process. We assume that the number of a firm's product lines follows a geometric Brownian motion, composed of a drift term determined by the firm's internal R&D activities and inter-firm knowledge diffusion, and a diffusion term arising from the uncertainty in R&D activities. This assumption provides two advantages. First, it allows us to abstract away from the firm entry and exit that are inevitably associated with integer-valued product lines in models like the Klette and Kortum (2004) model. As a result, we can also ignore inventor unemployment and off-the-job search, enabling us to focus on job-to-job transitions of inventors between existing firms and the associated knowledge diffusion. Second, it allows the state variables to be aggregated into a one-dimensional variable representing a firm's innovation productivity. Based on our assumption regarding the stochastic process of product lines, the competitive equilibrium can be characterized by a Hamilton-Jacobi-Bellman (HJB) equation for the joint value of firms and inventors per product line, and a Kolmogorov-Forward (KF) equation for the distribution of product lines, both with a single state variable.

The aggregation of the state variables into the innovation productivity variable not only alleviates the computational burden but also provides a clear linkage between microdata and model variables. Specifically, our dataset includes the number of patent citations per inventor at each establishment. By mapping this observable metric to the model's state variable, innovation productivity, we effectively align the microdata with the theoretical model.

We highlight the inefficiencies present in this economy by comparing the competitive equilibrium with the optimal allocation. Building on Nuno and Moll (2018), we characterize the optimal allocation determined by a social planner, who maximizes the utility of the representative household, in a manner comparable to the competitive equilibrium. The optimal allocation can be characterized by an HJB equation, modified from the equation for competitive equilibrium, and the same KF equation. By comparing the HJB equations for the competitive equilibrium and the optimal allocation, we identify two sources of inefficiency in this economy: (i) inefficiencies arising from creative destruction and competition and (ii) inefficiencies resulting from random search.

In our model, R&D investment activities and knowledge diffusion generate inefficiencies through an externality arising from random search, as well as the externality from creative destruction and competition[2]. The externality from random search can be further divided into two types: congestion externalities and hold-up problems. Hosios (1990) show that in a standard Diamond (1982), Mortensen (1982), and Pissarides (1985) (DMP) model, these externalities can be entirely offset by appropriately setting the bargaining power. However, in our model, which incorporates on-the-job search with multiple workers, no level of bargaining power can achieve the optimal allocation. The bargaining power that maximizes social welfare depends on how internal R&D activities and knowledge diffusion affect the economy through congestion externalities and hold-up problems.

We calibrate the parameters of our model using matched employer-employee data on

[2]We argue that the externality arising from random search in our model is a generalization of the externality present in the knowledge diffusion models from previous research, such as those by Lucas and Moll (2014) and Perla and Tonetti (2014).

German inventors. Specifically, we estimate the law of motion for inventor productivity. This equation is characterized by the inventor's job-to-job transition rate, as well as the inventor productivity at the focal firm and at other firms. All of these variables can be observed from our matched employer-employee-patent data. By estimating the law of motion for inventor productivity, we obtain the key parameter values that determine the magnitude of knowledge diffusion, and those associated with internal R&D. The calibrated model closely replicates the distribution of German inventor productivity, which is a non-targeted moment.

Using a calibrated model, we first analyze the impact of prohibiting non-compete clauses on economic growth and social welfare. In recent years, several countries have implemented or are planning to implement prohibitions or restrictions on non-compete clauses. For example, in April 2024, the Federal Trade Commission (FTC) banned all non-compete agreements in the US. In May 2023, the UK Government also announced plans to limit non-compete clauses to a maximum duration of three months. To analyze the impact of banning non-compete clauses on economic growth and social welfare, we examine the transition dynamics of our economy. When non-compete clauses are prohibited, the surplus that poaching firms can gain from posting a vacancy increases, leading to more vacancy postings. As a result, inventor mobility between firms increases. The increase in inventor mobility raises the economic growth rate through two channels. First, the improvement in allocative efficiency of inventors enhances growth driven by internal R&D. In our model, the misallocation of inventors arises due to (i) frictions in the labor market for inventors, (ii) idiosyncratic shocks, and (iii) the concavity of internal R&D technology. The increase in inventor mobility due to the prohibition of non-compete clauses mitigates this misallocation, thereby increasing the growth rate. Second, the increase in inventor mobility promotes knowledge diffusion between firms, further enhancing the economic growth rate. A higher growth rate leads to higher output during the transition period compared to an economy without policy changes. However, since firms allocate more resources to vacancy posting using final goods, consumption temporarily decreases. We interpret vacancy posting costs in our model to include not only the costs of search and hiring but also more generally various adjustment costs associated with inventors' job-to-job transitions. For example, these costs would include those associated with inventors and their families relocating, as well as the costs for inventors to build relationships with new research team members. Due to these higher costs, consumption temporarily declines, and the consumption-equivalent welfare in transition dynamics decreases by 0.38% compared to a scenario without the policy change. Furthermore, we find that optimal regulation of non-compete clauses achieves an allocation and social welfare very close to the social optimum.

Finally, we analyze the impact of the observed decline in inventor mobility on economic growth. Using a dataset from Germany, we find that the job-to-job transition rate for inventors decreased from 13.0% in 2001 to 5.1% in 2014. A recent study by Akcigit and Goldschlag (2023a) also finds that the hiring rate for inventors in the U.S. declined from 7% in 2000 to 3.5% in 2016. While our data is at the establishment level and their data is at the firm level—resulting in a consistently higher job-to-job transition rate in our data—both datasets indicate that inventor mobility halved over a similar period. We calibrate the transition path of matching efficiency so that the decline in inventor mobility observed in Germany aligns with the model's inventor mobility. As a result, we find that the decline in inventor mobility reduced the economic growth rate by 0.24 percentage points in the long run. Of this, 0.04 percentage points is due to the decrease in growth driven by internal R&D, caused by a deterioration in the allocative efficiency of inventors. The remaining 0.20 percentage points is due to a decline in knowledge diffusion between firms, resulting from the decrease in inventor mobility. Additionally, compared to the BGP in the scenario where matching efficiency did not decline, we find that the observed decline in inventor mobility reduced consumption-equivalent welfare by 3.2%.

This paper provides insights into a broader category of high-skilled workers, not limited to inventors. Our analysis focuses on inventors who have been granted patents since patent data allow us to observe knowledge creation. However, the mechanisms discussed here apply to various types of high-skilled workers, including engineers, scientists, entrepreneurs, managers, and executives. These workers contribute to knowledge diffusion by moving between firms, thereby promoting productivity growth. According to Federal Trade Commission (2023), approximately 30 million workers in the United States (about 18% of employees) are subject to non-compete clauses. Additionally, as Shi (2023) notes, around 64% of executives are bound by non-compete clauses, indicating that high-skilled workers are subject to such restrictions at a higher rate. Our research offers insights into how policies, such as the nationwide ban on non-compete clauses implemented in the U.S. in 2024, may impact firm and economic growth by altering the job flows of various high-skilled workers.

Related Literature

Our paper is most closely related to the literature on endogenous growth theory, particularly the diffusion of technology and knowledge, including Kortum (1997), Lucas and Moll (2014), Perla and Tonetti (2014), Akcigit et al. (2018), Buera and Oberfield (2020), Benhabib et al. (2021), Prato (2022), Crews (2023), Shi et al. (2024)[3]. Similar to our study, Benhabib et al. (2021) and Shi et al. (2024) analyze the interaction between internal R&D activity and knowledge diffusion between agents. Within this literature, our work is most closely aligned with Akcigit et al. (2018), Prato (2022), and Crews (2023), which model knowledge diffusion among inventors or high-skilled workers, though they do not explicitly account for firms and job-to-job transitions. In contrast, our research provides microfoundations for the movement of inventors across firms and the resulting knowledge diffusion, offering new insights into how labor market policies and changes in inventor mobility affect economic growth.

The idea that inventor mobility between firms facilitates knowledge diffusion is supported by numerous empirical studies. One of the early studies in this literature, Almeida and Kogut (1999), demonstrates that in regions with high levels of inventor mobility between firms, knowledge flows tend to be localized. Song et al. (2003) show that inventors who switch firms are more likely to base their work on ideas from their previous employer compared to other inventors at the hiring firm. Rosenkopf and Almeida (2003) analyze pairs of firms and find that firms with higher inventor mobility also experience greater subsequent knowledge flows. These pioneering studies have spurred further research into the relationship between inventor mobility and knowledge spillovers (Hoisl, 2007; Breschi and Lissoni, 2009; Singh and Agrawal, 2011; Kaiser et al., 2015; Rahko, 2017; Braunerhjelm et al., 2020). [4] Our paper is the first to explicitly model the fundamental idea from this empirically focused literature—that inventor mobility between firms leads to knowledge diffusion—and to integrate it into endogenous growth theory. Additionally, our research makes a novel empirical contribution to this literature. By comparing the patterns of job transitions and the accompanying changes in wages between inventors and general workers, we provide evidence that suggests inter-firm knowledge diffusion and the associated compensation.

Our paper is complementary to recent studies that analyze learning among workers within firms. Jarosch et al. (2021) and Herkenhoff et al. (2024) both develop frameworks for learning from colleagues within firms. Using matched employer-employee data, they both find significant knowledge spillovers among coworkers. Unlike these studies,

[3]Buera and Lucas (2018) provide a survey of this literature.

[4]Mawdsley and Somaya (2016) provide a review of this literature. A related area of study is the relationship between geography and knowledge diffusion. Early research by Jaffe et al. (1993) suggested a higher probability of cited patents originating from the same location as the citing ones. Breschi and Lissoni (2009) refined this approach by introducing inventor mobility as a control, revealing that the effect of spatial proximity on knowledge diffusion is reduced by more than half. This finding suggests that the critical role of geography in knowledge diffusion primarily stems from the infrequency of inventors relocating across regions.

our research focuses on inventors engaged in R&D and the implications for economic growth. Specifically, in the theoretical part, we construct an endogenous growth model, and in the empirical part, we analyze a novel dataset that links patents with matched employer-employee data.

Finally, our paper is related to the literature on frictional labor markets. Studies such as Schaal (2017), Elsby and Gottfries (2021), Lentz and Mortensen (2022), Bilal et al. (2022), and McCrary (2022) focus on firms employing multiple workers and on-the-job search. When the revenue function is not constant returns to scale and involves on-the-job search, solving the firm's problem becomes generally challenging due to the necessity of tracking the wage distribution within each firm. To address this challenge, Lentz and Mortensen (2022) and Bilal et al. (2022) consider an economy in which the joint value of firms and their workers is always maximized. We adopt their approach to analyze knowledge diffusion through the movement of inventors between firms. Building on Bilal et al. (2022), Bilal et al. (2023) present an endogenous growth model where, in an economy with on-the-job search, the productivity distribution of incumbent firms determines the productivity of new entrants, similar to Luttmer (2007). However, their model does not account for knowledge diffusion through the mobility of workers or inventors.

The rest of the paper proceeds as follows. Section 2 introduces the data and presents empirical findings. Section 3 presents the theoretical framework, starting with the competitive equilibrium, followed by the social planner's problem, and a discussion of externalities and policy instruments. Section 4 estimates the model using the data and discusses its fit. Section 5 uses the estimated model to examine the impact of the decline in inventor mobility and to evaluate the quantitative policy counterfactuals. Section 6 concludes.

2 Data and Empirical Findings

In this section, we investigate job flows of inventors between establishments using inventor biography data for Germany. Our analyses draw on two administrative data sets, "Linked Inventor Biography Data 1980–2014" (INV-BIO) and "Sample of Integrated Labor Market Biographies" (Stichprobe der Integrierten Arbeitsmarktbiografien – SIAB).[5]

The INV-BIO data set combines labor market biographies recorded in the German social security data (Integrated Employment Biographies — IEB) with patent register data from the European Patent Office (EPO). This data set tracks information about 152,350 inventors who registered their patents with the EPO from 1980 to 2014. The information

[5] More detailed information is presented in Online Appendix A.1.

includes their unique ID, age, gender, level of education, daily wage, and the number of citations received by the patents associated with each inventor in the EPO's records. The data also contain information about the establishments employing the inventors, such as the establishment ID, the total number of their employees, and the mean daily wage of their full-time employees. An important advantage over patent-based data sets used in previous studies (e.g., EPO patent data by Akcigit et al. (2018)) is that we can use social security information to track inventors' flows even when they are not creating patents.

The SIAB data set is a 2% random sample from IEB. It contains the same information about individuals and their employing establishments as INV-BIO, except for patent-related information. In the absence of patent data, we identify inventors in SIAB using a 3-digit occupation code, as described in Section 2.2. The sample covers 3,322,316 individuals from 1980 to 2019.

Because the two data sets cannot be merged, we use them separately for each analysis. When comparing the movement patterns of inventors and other workers in Section 2.2, we rely on SIAB, which includes both inventors and non-inventors. Otherwise, we use INV-BIO, a data set focused exclusively on inventor information.

2.1 Inventor Flows in INV-BIO

First, we adopt an approach similar to Haltiwanger et al. (2018) to characterize inventor flows using INV-BIO. We assign each establishment to a percentile rank based on either patent information or a productivity measure. We then compute the transition probabilities of inventor flows between these ranks.[6]

We use three measures as proxies for knowledge quality or productivity: (i) patent citation per inventor, (ii) the number of employees (establishment size), and (iii) the mean wage of full-time employees.[7] The first measure is based on the forward citations for patents that establishments have created. Measuring patent quality through forward citations is widely employed in the literature on patent creation (e.g., Pakes (1986); Hall et al. (2001); Akcigit et al. (2018)). Our measure for an establishment e in year t, z_{et}, is given by:

$$z_{et} = \sum_i \text{citations}_{it} \times \frac{n_{ie}}{n_i}$$

where citations_{it} denotes the count of forward citations that occur three years after year t

[6]Establishments can fall into different percentiles each year depending on the measure used. The ranks of the origin and destination establishments are determined by the measure from the previous year, preceding the movement of inventors.

[7]On-the-job search models with heterogeneous-productivity firms (e.g., Postel-Vinay and Robin (2002)) predict that more productive firms offer higher wages and attract more workers.

Table 1: Transition Probabilities of Inventor Flows

(A) Rank by Citation/Inventor

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.3	0.2	0.3	0.4	4.3
Origin	50-60	1.7	0.2	0.2	0.3	3.0
establishment	60-70	1.9	0.2	0.3	0.3	3.6
rank	70-80	2.2	0.2	0.2	0.4	4.2
	80-100	19.5	2.0	2.4	3.4	46.7

(B) Rank by Establishment Size

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.6	0.9	0.7	0.8	6.3
Origin	50-60	0.4	0.5	0.6	0.4	2.3
establishment	60-70	0.5	0.2	0.7	0.9	3.0
rank	70-80	0.6	0.3	0.4	1.3	4.8
	80-100	5.8	2.5	3.4	4.7	55.7

Notes: The detailed description appears below panel (C) on the next page.

for patent i, which was created by a team including inventors employed at establishment e. Note that the team developing the patent can consist of inventors from different establishments. n_{ie} represents the number of inventors at establishment e in the team, while n_i represents the total number of inventors in the team, including those affiliated with different establishments. We multiply citations$_{it}$ by n_{ie}/n_i to capture the contribution of inventors affiliated with establishment e.

Table 1 shows the transition probabilities of inventor flows from origin to destination percentile ranks.[8] The red-shaded cells highlight substantial movements from higher to lower ranks: the sums of these entries equal 33.7% in panel (A), 18.8% in panel (B), and 28.5% in panel (C). These shares indicate that many inventors move from higher-ranked establishments to lower-ranked ones.[9] Online Appendix B.1 shows that this pattern persists even when we restrict the sample to job flows accompanied by wage increases. This pattern does not appear in earlier work on worker flows. For example, Haltiwanger et al. (2018) construct transition probabilities based on firms' mean wages and find a higher

[8]Online Appendix B.1 reports the distribution of inventors for each measure. It reveals a notable concentration of inventors within specific establishments. Regardless of the measure, more than half of inventors are employed in establishments above the 80th percentile, and only about 10% are in establishments below the 50th percentile. This aligns with the finding by Akcigit and Goldschlag (2023b) that inventors are concentrated in large incumbents in the U.S.

[9]Another notable pattern is the large mass along the diagonals, especially in the bottom-right corner of each panel: 49.9% in panel (A), 60.8% in panel (B), and 50.8% in panel (C). This indicates that many inventors move within the same rank, particularly within the top rank, a pattern also documented in the worker-flow literature (e.g., Haltiwanger et al. (2018)).

Table 2: Inventor Flows across Establishments (cont.)

(C) Rank by Mean Wage

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.9	1.1	1.0	1.1	3.5
Origin	50-60	0.8	0.9	1.1	0.9	2.3
establishment	60-70	1.0	0.9	2.0	2.1	4.1
rank	70-80	1.4	1.0	1.7	3.6	7.3
	80-100	5.5	3.6	5.4	7.2	37.6

Notes: This table shows transition probabilities of inventor flows across establishment percentiles. Inventors who stay in the same establishment are excluded. The percentile rank in panel (A) is based on the three-year backward average of forward patent citation counts. Panel (B) is based on the number of employees, and panel (C) is based on the mean wage of full-time employees. Establishments can fall into different percentiles each year depending on the measure. The origin and destination ranks are determined using the measure from the previous year, prior to inventor moves. The sample covers 1980–2014. Each cell reports the proportion of total inventor flows in INV-BIO.

probability of flows toward higher ranks. The contrast suggests that the prevalence of flows to lower-ranked establishments is a distinctive feature of inventor mobility.

2.2 Inventor Flows in Comparison with Worker Flows

In this section, we compare inventor flows with worker flows. We use SIAB for this comparison because INV-BIO lacks information on non-inventor workers.

To identify inventors within SIAB, we use a 3-digit occupation code. We find that the majority of inventors in INV-BIO are affiliated with four occupations: research and development (20.2%), machine-building and operations (19.8%), mathematics, biology, and physics (19.1%), and mechatronics, energy, and electronics (18.8%). These occupations account for nearly 80% of the inventors in INV-BIO, so we treat SIAB workers in these categories as likely inventors.

Table 2 compares summary statistics across the two data sets. The mean daily wage of the identified inventors in SIAB lies between the average wage of all SIAB workers and that of inventors in INV-BIO. The share of female workers exhibits a similar ordering. These patterns suggest that our identified inventors include both actual inventors and some non-inventor workers. Consequently, the subsequent comparison between workers and identified inventors should be interpreted as conservative because attenuation bias likely attenuates the differences.

We estimate the following Probit model for job changers without unemployment spells:

$$P(D_{it} = 1) = \Phi(\beta_0 + \beta_1 I_{it} + \beta_2 X_{it}) \tag{1}$$

Table 2: Identified Inventors in SIAB and Inventors in INV-BIO

Summary statistics (1980 - 2014)		SIAB Workers	SIAB Identified inventors	INV-BIO Inventors
Daily wage, Euro	Mean	59.0	78.9	156.2
	S.D.	47.2	52.1	30.0
Age	Mean	38.7	38.4	42.4
	S.D.	12.9	12.4	9.0
Females, %		45.2	14.8	5.7
N of obs., thousand		21,344	2,871	420

Notes: This table compares the summary statistics between workers in SIAB and the inventors in INV-BIO. Identified inventors in SIAB are workers who work in the following four occupations: "research and development", "machine-building and operations", "mathematics, biology, and physics", and "mechatronics, energy, and electronics." The workers in the table include the identified inventors. The summary statistics are calculated using a pooled sample with daily wage, age, and gender filled in.

The indicator I_{it} equals one if individual i works in one of the four occupations in year t, and zero otherwise. D_{it} equals one if individual i moves from a more productive establishment to a less productive one in year t, and zero if the move is toward a more productive establishment. We focus on moves between establishments rather than ranks. To construct D_{it}, we use the number of employees or the mean wage as the productivity proxy. The control vector X_{it} includes age, age squared, gender, and educational attainment. To avoid the incidental parameter problem, we estimate the model without fixed effects.[10] The function Φ denotes the cumulative distribution function of the standard normal distribution.

Our coefficient of interest is β_1. A positive value indicates that inventors are more likely than other workers to move to less productive establishments. Standard errors are clustered by destination establishment and year to accommodate persistent establishment- or year-specific shocks.

Table 3 reports the estimation results. Columns (1) and (2) use establishment size and mean wage, respectively, to construct D_{it}. In both specifications, inventors are significantly more likely than other workers to move to less productive establishments. Columns (3) and (4) restrict the sample to job changers who receive wage increases, and the estimated coefficients on I_{it} remain positive and significant. Hence, many inventors move to less productive establishments even when their wages rise after the move.

To further examine the association between the direction of flows and wages, we run the following regression:

$$\log w_{it} - \log w_{it-1} = \beta_0 + \beta_1 D_{it} + \beta_2 I_{it} + \beta_3 D_{it} I_{it} + \beta_4 X_{it} + \alpha + \varepsilon_{it} \tag{2}$$

[10] A linear probability model with fixed effects delivers a significantly positive estimate of β_1 as reported in Table 3; see Online Appendix B.2.

Table 3: Estimation Result for Inventor Flows

	(1) $P(D_{it}=1)$				(2) $\Delta \log w_{it}$	
	Whole sample		Sample with wage ↑			
I_{it}	.077***	.036***	.052***	.012***	.017***	.021***
	(.004)	(.004)	(.004)	(.004)	(.005)	(.004)
D_{it}					-.078***	-.084***
					(.006)	(.005)
$D_{it} \times I_{it}$					.016***	-.002
					(.006)	(.006)
Control	√	√	√	√	√	√
Fixed Effects					√	√
Measure for D_{it}	Size	Mean wage	Size	Mean wage	Size	Mean wage
N	3,572,567	3,533,344	2,082,939	2,060,714	859,888	859,861
Adj. R^2	.019	.016	.005	.003	.13	.13

Notes: Control variables include age, age squared, gender, and educational attainment. Fixed effects include year, year × industry, and destination-establishment fixed effects. I_{it} equals one if individual i works in one of the four occupations ("research and development", "machine-building and operations", "mathematics, biology, and physics", and "mechatronics, energy, and electronics") in year t, and zero otherwise. D_{it} equals one if individual i moves to a less productive establishment in year t, and zero otherwise. The productivity measure is based on establishment size or mean wage in year $t-1$. The sample spans 1980–2019. Standard errors are clustered by year and destination establishment. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

The variable w_{it} represents the daily wage of individual i after a job change, while w_{it-1} represents the wage before the job change. The vector of fixed effects α includes year, year × industry, and destination establishment fixed effects. The definition of other variables remains the same as in the equation (1).

Columns (5) and (6) examine wage growth directly. Inventors experience wage gains of roughly 2% more than comparable workers when they change jobs. The coefficients on D_{it} are negative, showing that moves to less productive establishments are generally associated with smaller wage increases. However, in column (5) the interaction $D_{it} \times I_{it}$ is significantly positive, indicating that inventors experience less of a wage penalty than other workers when moving to less productive establishments.

Inventor mobility can plausibly generate these patterns through knowledge transfer. Inventors who leave high-productivity establishments bring ideas that can raise productivity elsewhere, making lower-ranked establishments willing to poach them and compensate them accordingly.

3 Model

In this section, we first introduce an endogenous growth model that highlights the role of inventor mobility and the resulting knowledge diffusion and characterize the competitive equilibrium. Next, we formulate the optimal allocation and examine the externalities present in this economy. The competitive equilibrium and optimal allocation introduced in this section will be numerically analyzed in Section 5.

Household and Production Technology

Time is continuous. The representative household is composed of a unit measure of individuals who supply inelastically L units of production labor and N units of inventors. The representative household consumes a single final good, and the utility at time 0 is given by

$$\int_0^\infty e^{-\rho t} \log C(t) dt.$$

where $\rho > 0$ is the discount rate. Each household is free to borrow or lend at interest rate $r(t)$ and $C(t)$ is the aggregate consumption at time t. We choose the numeraire so that $P(t)Y(t) = 1$ for all t, where $P(t)$ denotes the final good price and $Y(t)$ denotes the aggregate output[11]. The choice of numeraire implies $r(t) = \rho$ for all t[12].

There is a unit continuum of products to produce the final good, and the aggregate output is determined by the production function

$$\log Y(t) = \int_0^1 \log \left(z(\omega, t) l(\omega, t) \right) d\omega$$

where $l(\omega, t)$ is the quantity of product $\omega \in [0, 1]$ at time t, and $z(\omega, t)$ is the productivity of product ω at time t.

Production labor is the only factor in the production of each product. There is no friction in the production labor market. Labor productivity is the same for all products and is set equal to 1. This means that $l(\omega, t)$ represents both the output amount of product ω and the labor demand for the production of product ω. Each firm is the monopoly supplier of products. We assume that firms pay a marginally small operating cost $\epsilon \to 0$ before producing a product each period, like Acemoglu et al. (2018), to prevent price competition between the technology leader of the product and firms that produce that product in the past. Let $p(\omega, t)$ denote the price of product ω at t. Because the nominal output $P(t) Y(t)$

[11]The choice of the numeraire implies that $P(t)$ declines over time at the rate of economic growth.
[12]The derivation is in Appendix B.1.

is normalized to 1 and the final good technology is Cobb-Douglas, the revenue for each product line $p(t)\, l(\omega, t)$ is also equal to 1[13].

We assume that if a firm pays a cost $0 < \pi < 1$, it can copy and utilize a technology of other firms. As a result, π is the upper bound of the profit that a firm can obtain from a single product line[14][15]. A monopoly firm sets the price $p(\omega, t)$ to solve the following problem:

$$\max_{p(\omega,t)} \pi(\omega, t) = p(\omega, t)\, l(\omega, t) - w(t)\, l(\omega, t) \quad \text{s.t.} \quad \pi(\omega, t) \le \pi, \quad p(\omega, t)\, l(\omega, t) = 1$$

Then, the price chosen by the firm is $p(\omega, t) = w(t)/(1-\pi)$ and the labor demand is $l(\omega, t) = (1-\pi)/w(t)$. From the labor demand and labor market clearing condition $\int_0^1 l(\omega, t) d\omega = L$, the production wage is given by $w(t) = (1-\pi)/L$. Therefore, we have $p(\omega, t) = 1/L$ and $l(\omega, t) = L$.

Innovation Technology

The quality of each product is determined by the total number of innovations that have been implemented for that product in the past:

$$z(\omega, t) = \lambda^{m(\omega,t)}$$

where $m(\omega, t)$ is the number of innovations made to product ω up to time t, and $\lambda > 1$ is the step size of innovation, assumed to be the same across all products. All products face the same rate of creative destruction, denoted by $\delta(t)$, which is endogenously determined by the aggregation of innovation activities. By the law of large numbers, we have:

$$\log Y(t) = \log \lambda \int_0^t \delta(\tau)\, d\tau + \log L \tag{3}$$

Therefore, the economic growth rate is proportional to the creative destruction rate:

$$\frac{d}{dt} \log Y(t) = \delta(t) \log \lambda \tag{4}$$

[13]The derivation is in Appendix B.2

[14]We consider the size of π to be exogenously determined by factors such as the strength of patent enforcement or taxation on sales.

[15]In a quality ladder model with a Cobb-Douglas aggregator, it is common to assume Bertrand competition between the firm currently producing the product and the firm that previously produced it. In this case, when the step size of innovation is constant, the instantaneous profit for each product line is equal across all product lines, as in the result derived here.

Each firm supplies a continuum of products, denoted by k, and employs a continuum of inventors, denoted by n. The mass of a firm's product lines k changes for three reasons:

(i) Internal R&D: The mass of a firm's product lines k changes as a result of internal R&D activities:

$$d\log k\big|_{R\&D} = \left\{\mu\left(n/k\right) - \frac{\sigma^2}{2}\right\} dt + \sigma dW\left(t\right)$$

where $dW\left(t\right)$ is a Wiener process, and $\mu(\cdot)$ is increasing, concave, and $\mu(0) \geq 0$, and $\sigma > 0$. Under the assumption on $\mu(\cdot)$, the drift of R&D outcome per inventor $\mu\left(n/k\right) k/n$ is increasing in k/n. From this observation, we refer to the ratio of knowledge capital to inventor measure k/n as inventor productivity. The stochastic process of k due to internal R&D assumed here is reminiscent of the assumption in Klette and Kortum (2004) as the expected R&D outcome $\mu\left(n/k\right) k$ exhibits constant returns to scale in the knowledge capital k and the mass of inventors n. Klette and Kortum (2004) assume that a firm's knowledge capital facilitates innovation, and that knowledge capital can be summarized by the number of a firm's product lines. We also follow this assumption, and in the following, we refer to k as either knowledge capital or the mass of product lines, interchangeably. The difference between ours and Klette and Kortum (2004) is that in our model, k takes a continuous value, whereas in their model, k takes an integer value. Furthermore, as we will discuss later, unlike the model of Klette and Kortum (2004), there is a cost to adjusting the mass of inventors n due to labor market frictions.

(ii) Knowledge diffusion due to inventor job flows: When the mass of inventors at a (k, n)-firm increases by 1% due to the inflow of inventors from a (k', n')-firm, the knowledge capital of the (k, n)-firm increases by $\hat{\alpha}(k/n, k'/n')$%:

$$d\log k\big|_{\text{from }(k',n')\text{-firm}} = \hat{\alpha}(k/n, k'/n')\; d\log n\big|_{\text{from }(k',n')\text{-firm}}$$

where $\hat{\alpha}(k/n, k'/n')$ is an exogenous function of the inventor productivity of both the poaching firm and the poached firm. We assume that $\partial\left(\hat{\alpha}(k/n, k'/n')\right)/\partial\left(k/n\right) < 0$ and $\partial\left(\hat{\alpha}(k/n, k'/n')\right)/\partial\left(k'/n'\right) > 0$. In other words, (i) the more productive the inventors at a poached firm are, or (ii) the less productive the inventors at a poaching firm are, the more knowledge capital the poaching firm will gain when hiring these inventors.

(iii) Creative destruction by other firms: Each product line of a firm faces the possibility that other firms will innovate on it. If a firm has k product lines, it will lose product lines at a rate of $\delta\left(t\right) k$:

$$d\log k\big|_{\text{Destruction}} = -\delta\left(t\right) dt$$

Matching Technology and Bargaining

Firms and inventors meet in a single frictional labor market, and search is random. We focus on on-the-job search and assume that unemployment benefits are so low that there is no voluntary unemployment. As in Kaas and Kircher (2015), we assume a constant returns to scale vacancy cost with respect to vacancy and firm size: A firm pays a cost $c\left(\hat{v}/k\right)k$ to post $\hat{v}$ vacancies. The vacancy cost is in terms of final good. The cost function $c\left(\cdot\right)$ is increasing and concave, and satisfies $c(0) = 0$ and $c'(0) = 0$[16].

An aggregate matching function $m(N, V)$ determines the flow of meetings between firms and inventors as a function of the measure of inventors, N, and the total measure of vacancies posted by firms V. Each firm meets an inventor at a Poisson rate proportional to the number of vacancies it posts. Each inventor meets a firm at a Poisson rate identical across all inventors. We assume that m exhibits constant returns to scale in (N, V) and decreasing returns to scale in either N or V separately. Let $\theta = V/N$ denote labor market tightness. Then, the Poisson rate at which a unit of vacancy meets an inventor is $q(\theta) \equiv m(N, V)/V$, and the rate at which an inventor meets a firm is $\theta q(\theta) = m(N, V)/N$.

Our assumptions on bargaining and vacancy posting follow Lentz and Mortensen (2022). When a firm and an inventor meet, a match is formed if the match surplus among the three parties (the poaching firm, targeted firm, and inventor) is positive. In this case, the three parties bargain over the match surplus. As a result of bargaining, the share $\beta \in (0, 1)$ of the surplus is allocated to the coalition of the targeted firm and inventor, and the residual is allocated to the poaching firm. We do not specify how the surplus is allocated between the targeted firm and inventor. In addition, we assume that the firm posts the privately efficient amount of vacancies, which is the one that maximizes the sum of the values of the firm and its inventors[17].

Under these assumptions, firms' and inventors' decisions are privately efficient, as if the firm and incumbent inventors maximize their total value. Because the inventors are the same ex-ante, the state variables of the total value function are only two: inventor employment n and knowledge capital k.

[16] We consider vacancy costs not just as the cost of recruiting but more broadly as labor adjustment costs. The management literature on labor turnover often emphasizes the significant transaction costs associated with labor mobility. Not only is hiring new employees costly, but training them is as well. Moreover, the relationships and informal communication structures within the firm are disrupted.

[17] Hawkins (2015) discusses how vacancy posting becomes privately optimal under the assumption that firms can commit to wage contracts. Also, under the assumption that firms cannot commit to wage contracts, Bilal et al. (2022) provide an argument where jointly efficient vacancy posting is a result of a particular communication protocol between the firm and its workers.

Distribution of Firms and Product Lines

To formally characterize the equilibrium, we need to introduce the distribution of firms and product lines. Let $g(k,n,t)$ denote the measure of firms in the (k,n)-space. Then, the measure of product in the (k,n)-space is given by $kg(k,n,t)$. As we will confirm later, the state variable of a firm is reduced to the number of inventors per product line, $x \equiv \frac{n}{k}$. Anticipating this, let $f(x,t)$ denote the measure of product in the x-space, which is defined as:

$$f(x,t) = \int_0^\infty \int_0^\infty kg(k,n,t)\,\boldsymbol{\delta}\left(\frac{n}{k} - x\right) dk dn$$

where $\boldsymbol{\delta}(x)$ is the Dirac delta function. Since the total mass of the product is one, we have:

$$1 = \int_0^\infty f(x,t)dx. \tag{5}$$

Also, since all inventors are employed, we have:

$$N = \int_0^\infty xf(x,t)dx \tag{6}$$

Let v denote the amount of vacancy per product line, where $v = \hat{v}/k$. The total amount of vacancies is given by:

$$V(t) = \int_0^\infty v(x,t)\, f(x,t)\, dx \tag{7}$$

To characterize the equilibrium under random search, it is useful to define the distributions weighted by the mass of inventors and the mass of vacancies. Let $g_n(k,n,t)$ denote the inventor-weighted firm distribution in the (k,n)-space, and $g_{\tilde{v}}(k,n,t)$ denote the vacancy-weighted firm distribution in the (k,n)-space:

$$g_n(k,n,t) = \frac{ng(k,n,t)}{N}$$
$$g_{\tilde{v}}(k,n,t) = \frac{\tilde{v}(k,n,t)\, g(k,n,t)}{V(t)}$$

Analogously, let $f_x(x,t)$ denote the inventor-weighted product density in the x-space, and $f_v(x,t)$ denote vacancy-weighted product density in the x-space:

$$f_x(x,t) = \frac{xf(x,t)}{N}$$

$$f_v(x,t) = \frac{v(x,t)\, f(x,t)}{V(t)}$$

In addition, we express the cumulative distribution function (c.d.f.) in uppercase letters. For example, the c.d.f. of the inventor-weighted firm in the (k,n)-space is denoted by $G_n(k,n,t)$.

HJB Equation

The decision regarding the amount of vacancies posted and whether inventors actually move after being matched can be characterized by the HJB equation. Let $\Omega(k,n,t)$ denote the joint value of a firm with product mass k and its n inventors at time t, which satisfies the HJB equation:

$$\begin{aligned}
\rho\Omega(k,n,t) - \frac{\partial}{\partial t}\Omega(k,n,t) = \max_{\hat{v}\geq 0} \; & \underbrace{\pi k}_{\text{Profit}} - \underbrace{c\left(\hat{v}/k\right)k}_{\text{Search cost}} + \underbrace{\left\{\mu\left(n/k\right) - \delta\left(t\right)\right\} k\Omega_k(k,n,t) + \frac{\sigma^2}{2}k^2\Omega_{kk}(k,n,t)}_{\text{R\&D}-\text{Creative Destruction}} \\
& + \underbrace{\left(1-\beta\right) q(\theta\left(t\right))\hat{v} \int \hat{M}\left(k/n, k'/n', t\right) dG_n(k',n',t)}_{\text{Poaching}} \\
& + \underbrace{\beta\theta\left(t\right) q\left(\theta\left(t\right)\right) n \int \hat{M}\left(k'/n', k/n, t\right) dG_{\tilde{v}}(k',n',t)}_{\text{Being poached}},
\end{aligned}$$

where the match surplus $\hat{M}\left(k/n, k'/n', t\right)$ is given by

$$\hat{M}\left(k/n, k'/n', t\right) = \max\left[\Omega_n(k,n,t) + \hat{\alpha}\left(k/n, k'/n'\right) k/n\Omega_k(k,n,t) - \Omega_n\left(k',n',t\right), 0\right].$$

The intuition behind the HJB equation is as follows. The first term on the right-hand side, πk, represents the current profit. The second term, $c\left(\hat{v}/k\right)k$, represents the search cost. The third and fourth terms capture the change in the value due to the internal R&D activities and the loss of product lines through creative destruction by other firms.

The fourth term represents the change in the value from poaching new inventors from other firms. The firm posts $\hat{v}$ vacancies, and each vacancy matches with an inventor at the rate of $q(\theta)$. The inventors from other firms are matched according to the inventor-weighted distribution G_n. If a match occurs and the match surplus is positive, the inventor moves to the poaching firm. In this case, the poaching firm obtains a $\left(1-\beta\right)$ share of the match surplus.

The fifth term represents the change in the value when the firm's own inventors are poached by other firms. The firm has n inventors, and each inventor matches with a vacancy at the rate of $\theta(t)\, q(\theta(t))$. These inventors are matched with other firms according to the vacancy-weighted distribution $G_{\tilde{v}}$. The firm that loses the inventor receives a β share of the match surplus.

The matching surplus is defined as the gain of the firm that poaches an inventor minus the loss incurred by the firm from which the inventor is poached. The gain for the poaching firm consists of (i) the increase in value due to the rise in the mass of inventors and (ii) the increase in value due to knowledge diffusion. The loss for the firm from which the inventor was poached is (iii) the decrease in value due to the reduction in the mass of inventors.

From the HJB equation, we can confirm that the value function is homogeneous of degree one in (k, n). Therefore, the HJB equation can be rewritten in terms of the value per product line, with the number of inventors per product line, x, as the single state variable. Let $S(x, t)$ denote the joint value of a product line and its x inventors at time t:

$$S(x,t) = S\left(\frac{n}{k}, t\right) = \Omega(k,n,t)/k$$

The firm-level marginal value of inventor and knowledge capital can be written using $S(x, t)$ as

$$\begin{aligned} \Omega_n(k,n,t) &= S_x(x,t), \\ \Omega_k(k,n,t) &= S(x,t) - xS_x(x,t), \end{aligned}$$

and the second derivative of firm-level joint value with respect to knowledge capital is expressed as

$$\Omega_{kk}(k,n,t) = \frac{1}{k} x^2 S_{xx}(x,t).$$

When a firm hires a marginal unit of inventor, the number of inventors per product line increases by $1/k$ in all k units of product lines. As a result, the firm-level marginal value of inventors $\Omega_n(k, n, t)$ equals $k \times 1/k \times S_x(x, t)$. In addition, when a firm's knowledge capital increases by one unit, the firm's product line increases by one unit, but the number of inventors per product line decreases by n/k. Therefore, the firm-level marginal value of knowledge capital $\Omega_k(k, n, t)$ equals the value of one product line $S(x, t)$ minus $n/k \times S_x(x, t)$.

Then, we can rewrite the HJB equation at firm level into the HJB equation at the product level:

$$\rho S(x,t) - \frac{\partial}{\partial t} S(x,t) = \max_{v \geq 0} \Big\{ \pi - c(v) + \big[\mu(x) - \delta(t)\big]\left[S(x,t) - x S_x(x,t)\right] + \frac{\sigma^2}{2} x^2 S_{xx}(x,t) + \left(1-\beta\right) q\big(\theta(t)\big) v \int M(x,x',t)\, dF_x(x',t) + \beta \theta(t)\, q\big(\theta(t)\big) x \int M(x',x,t)\, dF_v(x',t) \Big\}, \tag{8}$$

where the match surplus $M(x,x',t)$ is given by

$$M(x,x',t) = \max\left[S_x(x,t) + \alpha(x,x')\, x^{-1}\left\{S(x,t) - x S_x(x,t)\right\} - S_x(x',t), 0\right]. \tag{9}$$

Here $\alpha(x,x') \equiv \hat{\alpha}(1/x, 1/x')$ maps the firm-level knowledge diffusion function into the inventor-per-product state variable.

From (8), the first-order condition for the firm's vacancy decision gives

$$c'(v(x,t)) = \left(1-\beta\right) q(\theta(t)) \int M(x,x',t)\, dF_x(x',t) \tag{10}$$

Therefore, the vacancy policy is increasing in the share of matching surplus for poaching firm $\left(1-\beta\right)$, matching probability $q(\theta(t))$, and the expected match surplus $\int M(x,x',t)\, dF_x(x',t)$.

KF Equation

To complete the description of the environment, we need to specify the evolution of the inventor productivity distribution at the product line level, $f(x,t)$. First, define the poaching indicator function $\mathbb{1}_P$ that takes 1 if the poaching succeeds and takes 0 otherwise:

$$\mathbb{1}_P(x,x',t) = \begin{cases} 1 & \text{if} \quad M(x,x',t) > 0 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Standard arguments give the KF equation, a partial differential equation governing the evolution of the distribution:

$$\frac{\partial}{\partial t} f(x,t) = \left\{ \frac{q(\theta(t)) v(x,t)}{x} \int_0^\infty \mathbb{1}_P(x,x',t) \alpha(x,x')\, f_x(x',t) dx' + \mu(x) - \delta(t) \right\} f(x,t)$$

$$
-\frac{\partial}{\partial x}\left[\left\{\begin{array}{c} \frac{q(\theta(t))v(x,t)}{x}\int_0^\infty \mathbb{1}_P(x,x',t)\left\{1-\alpha\left(x,x'\right)\right\} f_x(x',t)dx' \\ -\theta\left(t\right)q(\theta\left(t\right))\int_0^\infty \mathbb{1}_P(x',x,t)f_v(x',t)dx'-\mu\left(x\right)+\delta\left(t\right)\end{array}\right\} xf(x,t)\right]
$$

$$
+\frac{\partial^2}{\partial x^2}\left[\frac{\sigma^2}{2}x^2 f(x,t)\right] \tag{12}
$$

The first term captures changes in the mass of the product lines with x mass of inventor per product line. The second term captures changes in the mass of inventors within product lines with x mass of inventor per product line. The third term captures the effect of idiosyncratic shocks on the distribution.

Creative Destruction Rate

The rate of creative destruction for each product is given by

$$
\delta\left(t\right)=\underbrace{\int\mu\left(x\right)dF(x,t)}_{\text{Internal R\&D}}+\underbrace{N\theta\left(t\right)q\left(\theta\left(t\right)\right)\int\int\mathbb{1}_P(x,x',t)\alpha\left(x,x'\right)x^{-1}dF_x(x',t)dF_v(x,t)}_{\text{Knowledge Diffusion}}. \tag{13}
$$

The first term represents the sum of internal R&D outcomes across all firms. The second term represents the total net diffusion of knowledge through all inventor job flows. Therefore, because the economic growth rate is proportional to the creative destruction rate, the contribution to economic growth can be decomposed into that driven by internal R&D and by knowledge diffusion.

Resource Constraint

The final good output is either consumed by the representative household or used to create vacancies. The total expenditure on vacancy posting is given by $\int_0^\infty c\left(v\left(x,t\right)\right)dF(x,t)$, and since the nominal output $P\left(t\right)Y\left(t\right)$ is normalized to 1, the amount of the final good used for creating vacancies is $Y(t)\int_0^\infty c\left(v\left(x,t\right)\right)dF(x,t)$. Therefore, the following resource constraint must be satisfied:

$$
C(t)+Y(t)\int_0^\infty c\left(v\left(x,t\right)\right)dF(x,t)=Y(t) \tag{14}
$$

Definition of Equilibrium

We now define the equilibrium.

Definition 1. *A competitive equilibrium* consists of a sequence of joint value, vacancy policy, poaching indicator function, inventor productivity distribution, creative destruction rate, and labor market tightness

$$\left[\left\{S(x,t), v(x,t), \{\mathbb{1}_P(x,x',t)\}_{x'\in(0,\infty)}, f(x,t)\right\}_{x\in(0,\infty)}, \delta(t), \theta(t)\right]_{t\in[0,\infty)}$$

such that, given the initial inventor productivity distribution $f(x,0)$ that satisfies $1 = \int_0^\infty f(x,0)dx$ and $N = \int_0^\infty x f(x,0)dx$,

1. the joint value $S(x,t)$ satisfies the HJB equation (8),
2. the vacancy posting is optimal, so that $v(x,t)$ satisfies (10),
3. Given a match, the inventor moves if the match surplus is positive, so that the poaching indicator function $\mathbb{1}_P(x,x',t)$ is given by (11),
4. the inventor productivity distribution $f(x,t)$ follows the KF equation (12),
5. the creative destruction rate $\delta(t)$ satisfies (13), and
6. the labor market tightness $\theta(t)$ satisfies (7).

The steady-state equilibrium is the one where all variables are constant over time.

Decomposition of Firm-level Growth

In this model, we can decompose the factors contributing to the growth of firms' knowledge capital, the growth of the mass of employed inventors, and the growth of inventors' productivity within firms. The growth rate of knowledge capital in a firm with x inventors per product line is expressed as follows:

$$\begin{aligned} d\log k(x,t) = & \underbrace{\left(\mu(x) - \delta(t) - \frac{\sigma^2}{2}\right)dt + \sigma dW(t)}_{\text{Internal R\&D}-\text{Creative Destruction}} \\ & + \underbrace{\frac{q(\theta(t))v(x,t)}{x}\int_0^\infty \mathbb{1}_P(x,x',t)\alpha(x,x')f_x(x',t)dx'dt}_{\text{Knowledge Gain from Poaching}} \end{aligned} \tag{15}$$

The first line on the right-hand side captures how firms' knowledge capital grows through internal R&D and decreases due to creative destruction by other firms. The second line reflects the increase in knowledge capital through knowledge diffusion driven by inventor inflows.

The growth rate of inventor employment of a firm with x inventors per product line is given by

$$d\log n\left(x,t\right)=\underbrace{\frac{q(\theta\left(t\right))v\left(x,t\right)}{x}\int\mathbb{1}_{P}(x,x',t)dF_{x}(x',t)dt}_{\text{inventor inflow rate}}\\ \underbrace{-\,\theta\left(t\right)q(\theta\left(t\right))\int_{0}^{\infty}\mathbb{1}_{P}(x',x,t)f_{v}(x',t)dx'dt}_{\text{inventor outflow rate}},\tag{16}$$

which is expressed as the difference between the inventor inflow rate and the inventor outflow rate.

The growth rate of inventor productivity is given by $d\log x^{-1}=d\log k-d\log n$, and we use this relationship in Section 4 to estimate the parameter values.

Advantage of Assuming Continuum Mass of Product lines

In our model, the number of product lines is treated as a continuous variable and follows a geometric Brownian motion. In contrast, the model proposed by Klette and Kortum (2004) and those based on it assume that the number of product lines a firm owns is an integer. In this section, we argue that the feature of our model significantly simplifies the problem.

Our model satisfies not only the weak version of Gibrat's law but also the strong version, which is crucial for simplifying the problem. In our model, firm size, whether measured by production worker employment or sales, is proportional to the number of product lines k. As shown in (15), the growth rate of k is independent of k when conditioned on inventor productivity. Therefore, in our model, not only is the mean growth rate of a firm independent of firm size (a weak version of Gibrat's law), but the entire distribution of growth rates is also independent of firm size (a strong version of Gibrat's law).

In contrast, the models proposed by Klette and Kortum (2004) and those based on it, such as Atkeson and Burstein (2019), do not satisfy the strong version of Gibrat's law. In these models, the number of product lines takes an integer value, and a firm acquires and loses product lines at Poisson arrival rates proportional to its size. In the absence of other types of shocks, the mean growth rate of a firm remains independent of firm size, satisfying a weak version of Gibrat's law. However, averaging within the firm implies that the variance of firm growth is inversely proportional to firm size. Consequently, the entire distribution of growth rates depends on firm size, violating the strong version of Gibrat's law[18].

A strong version of Gibrat's law simplifies the model when R&D resources exhibit stickiness. In existing models of firm dynamics and innovation, firms are typically able to adjust their R&D resources freely, making R&D resources unnecessary as state variables. However, when firms face constraints in adjusting these resources, at least two state variables—firm size and R&D resources—are required. Despite this constraint, if the strong version of Gibrat's law holds, the ratio of firm size to R&D resources (x in our model) becomes a sufficient state variable to describe the firm's dynamics.

In addition, our formulation has the advantage of abstracting from the extensive margin of firms and inventors. If product lines are modeled as continuous variables following a geometric Brownian motion, it is possible to construct an equilibrium where no firms enter or exit the market. This implies that we do not need to consider inventor unemployment caused by firm exits. However, if the number of a firm's product lines is an integer and the firm stochastically loses product lines, all firms will eventually lose their entire product lines and exit the market. Consequently, considering the entry and exit of firms becomes inevitable. In our research, we deliberately ignore firm entry and exit, inventor unemployment, and off-the-job search, and instead focus on job-to-job transitions among inventors and the resulting knowledge diffusion.

Social Optimum

In this section, we define the social planner's allocation and discuss the externalities present in this economy. The social planner chooses the path

$$\left[Y(t), \delta(t), \theta(t), \left\{v(x,t), f(x,t), \{\mathbb{1}_P(x,x',t)\}_{x'\in(0,\infty)}\right\}_{x\in(0,\infty)}\right]_{t\in[0,\infty)}$$

to maximize

$$\int_0^\infty e^{-\rho t}\left[\log Y(t) + \log\left\{1 - \int_0^\infty c\left(v\left(x,t\right)\right) dF(x,t)\right\}\right] dt \tag{17}$$

subject to (4) , (13), (7), and (12), given the initial output $Y(0)$ and initial density $f(x,0)$.

We now define the social planner's allocation, which is structured to be comparable with the competitive equilibrium (Definition 1). The derivation is in Appendix B.3.

[18]Stanley et al. (1996) document that if each firm consists of a number of equal sized units and these units growth rates are independent, then the negative correlation between firm size and the standard deviation of the growth rate becomes too large compared to the data. They argue that the small correlation indicates the presence of strong, positive correlations among the firm's units.

Definition 2. *A social optimum* consists of a sequence of joint value, vacancy policy, poaching indicator function, inventor productivity distribution, creative destruction rate, labor market tightness, and instantaneous social gain from a product line

$$\left[\left\{S(x,t), v(x,t), \{\mathbb{1}_P(x,x',t)\}_{x'\in(0,\infty)}, f(x,t)\right\}_{x\in(0,\infty)}, \delta(t), \theta(t), \pi(t)\right]_{t\in[0,\infty)}$$

such that, given the initial inventor productivity distribution $f(x,0)$ that satisfies $1 = \int_0^\infty f(x,0)dx$ and $N = \int_0^\infty xf(x,0)dx$,

1. the joint value $S(x,t)$ satisfies the HJB equation

$$\begin{aligned}\rho S(x,t) - \frac{\partial}{\partial t}S(x,t) = \max_{v\geq 0} \; & \pi(t) - c(v) + \left\{\mu(x) - \delta(t)\right\}\{S(x,t) - xS_x(x,t)\} + \frac{\sigma^2}{2}x^2 S_{xx}(x,t) \\ & + q(\theta(t))\, v \int M(x,x',t)\, dF_x(x',t) \\ & + \theta(t)\, q(\theta(t))\, x \int M(x',x,t)\, dF_v(x',t), \\ & + v\theta(t)\, q'(\theta(t)) \int\int M(x'',x',t) dF_x(x',t) dF_v(x'',t)\end{aligned} \tag{18}$$

2. the vacancy posting policy $v(x,t)$ satisfies

$$\begin{aligned}c'(v(x,t)) = {} & q(\theta(t)) \int M(x,x',t)\, dF_x(x',t) \\ & + \theta(t)\, q'(\theta(t)) \int\int M(x'',x',t) dF_x(x',t) dF_v(x'',t)\end{aligned}$$

3. the poaching indicator function $\mathbb{1}_P(x,x',t)$ satisfies (11),

4. the inventor productivity distribution $f(x,t)$ follows the KF equation (12),

5. the creative destruction rate $\delta(t)$ satisfies (13),

6. the labor market tightness $\theta(t)$ satisfies (7), and

7. the instantaneous social gain from a product line $\pi(t)$ is determined so that

$$\underbrace{\frac{\ln\lambda}{\rho}\left[1 - \int_0^\infty c(v(x,t))\, f(x,t)dx\right]}_{\text{Gain in Consumer Surplus from Innovation}} = \underbrace{\int_0^\infty [S(x,t) - S_x(x,t)\, x]\, f(x,t)dx.}_{\text{Expected Marginal Social Value of Innovation}} \tag{19}$$

The stationary version of the social planner's allocation is the one where all variables are constant over time.

By comparing the conditions of competitive equilibrium and social planner's allocation, we can see that there are two sources of externalities: innovation and search friction. Below, we will explain these in turn.

The externality associated with innovation consists of two effects. First, even though firms can raise the profit from products where they created better technology, they will be able to appropriate only a portion of the gain in consumer surplus created by the innovation. This first effect is called the appropriability effect. Second, the innovator is stealing the business of the previous producer by replacing it. This second effect is called the business stealing effect[19]. The externality arising from these two effects causes the discrepancy between the private and social instantaneous gain from a product line, π. The instantaneous social gain from a product line is determined so that (19) is satisfied, which equates the gain in consumer surplus from innovation and the expected social marginal value of innovation.

The externality associated with search friction also consists of two effects. First, the increase in vacancy by one firm makes the matching of the other firms difficult due to congestion. This effect is captured by the fourth line of (18). Because firms do not take into account this congestion externality, this situation leads to too many vacancies compared with the socially optimal level.

The second inefficiency that arises from search friction is the hold-up problem. As in the standard Diamond-Mortensen-Pissarides (DMP) model, our model treats vacancy posting as a firm's investment. Matches are formed because of the firms' investment, and inventors do not incur any costs. Thus, all returns from the match should be paid to the firms to ensure an efficient level of investment. However, by the time the inventor and the firm engage in Nash bargaining, the investment costs are already sunk. As a result, (i) when the firm hires an inventor, it can collect only a $(1-\beta)$ share of the surplus in the second line of (8) and (ii) when the firm loses an inventor, the firm can collect only β share of the surplus in the third line of (8). This inefficiency, due to the firm's imperfect appropriation of the surplus, leads to too few vacancies compared with the socially optimal level[20].

In our model, R&D activity and knowledge diffusion affect inefficiency through congestion externality and hold-up problems. All the terms associated with congestion externality (the fourth line of (18)) and the hold-up problem (the difference in the second

[19]See Chapter 12 of Acemoglu (2008) for more detailed explanation about the typical externalities that arise from the Schumpeterian growth model.

[20]The recent paper by Fukui and Mukoyama (2024) discusses the inefficiency of on-the-job search.

and third lines between (8) and (18)) contain match surplus $M(x, x', t)$. From (9), we know that match surplus captures the net surplus that arises from changes in inventor allocation and from knowledge diffusion. Therefore, inefficiencies arise in our economy as a result of the interaction between innovation activities and the externalities stemming from random search.

Bargaining Power as a Policy Instrument

We consider the bargaining power β as a policy instrument. Achieving the socially optimal allocation requires policies that are contingent on inventors' productivity, but such policies are rare in practice. As a second-best approach, we focus on determining the optimal bargaining power. Bargaining power can be influenced by the regulation of non-compete clauses. For example, Shi (2023) models a contractual environment involving non-compete clauses. In her model, the joint value of the firm and the incumbent worker is maximized—similar to our framework—with the enforceability of non-compete clauses increasing the share of surplus captured by the firm and its incumbent worker. Furthermore, in the standard DMP model, optimal bargaining power can fully eliminate inefficiencies caused by congestion externalities and hold-up problems. This implies that a significant portion of these inefficiencies could potentially be mitigated in our model as well. For these reasons, we consider bargaining power to be the most natural and compelling policy within our framework.

Our model captures several effects of the strength of non-compete clauses on firms' decision-making. First, the strengthening of non-compete clauses reduces the share of the match surplus that a poaching firm can obtain $(1 - \beta)$, thereby decreasing the incentive to hire new inventors. Second, as a result of the first effect, the strengthening of non-compete clauses increases the incentive to hire inventors because it makes it harder for other firms to poach them. Third, the strengthening of non-compete clauses increases the share of the match surplus that a firm receives when its inventor is poached by another firm (β), thus increasing the incentive to post more vacancies and accumulate more knowledge capital and inventors.

Relationship with Existing Endogenous Growth Model with Knowledge Diffusion

In this section, we discuss the differences between existing endogenous growth models that address knowledge diffusion and our model.

First, the way match surplus is distributed differs from that in many models in the existing literature. Early models, such as Lucas and Moll (2014) and Perla and Tonetti

Table 4: Functional Forms

Function	Description
$c(v) = \frac{\overline{c}}{\phi+1} v^{\phi+1}$	Vacancy cost function
$q(\theta) = \overline{q}\theta^{\eta-1}$	The rate at which a vacancy meets an inventor
$\mu(x) = \overline{\mu}x^{\epsilon}$	Internal R&D function
$\alpha(x, x') = \overline{\alpha}\frac{x}{x'}$	Knowledge capital diffusion function

(2014), as well as models with micro-foundations for inventors like Akcigit et al. (2018) and Prato (2022), share the feature with our model that agents randomly match, leading to the knowledge diffusion between them. However, in these existing models, only the agent receiving the knowledge gains the surplus generated from the match. Consequently, these models do not internalize the effects of others learning from their own knowledge or technology.

In Shi et al. (2024) and Benhabib et al. (2021), as in our model, the surplus generated from a match is allocated between the two agents in fixed proportions based on their respective bargaining power. However, the bargaining power and its relationship to economic structures or policies differ between their models and ours. While they associate bargaining power with the strength of patent enforcement in the real economy, our model—where knowledge is transmitted through inventor mobility—links bargaining power to labor market structures or policies concerning inventors, such as the enforceability of non-compete clauses.

Second, another novel aspect of our model is that it incorporates knowledge diffusion between firms as a result of inventor job flows. While models with micro-foundations for inventors, such as those by Akcigit et al. (2018) and Prato (2022), do not account for inventor mobility between firms, our model explicitly includes this mechanism. By considering firms' learning processes through inventor mobility, our model enables analysis that integrates microdata on inventor job flows and patents.

4 Estimation

In this section, we estimate the model using the German matched employer-employee-patent data. We describe the assumptions regarding the functional form, the estimated parameters, the moments that we target, and the estimation method.

Table 5: Estimated Parameters

Parameter	Value	Description
		A. Externally Set or Normalized
ρ	0.004	Discount rate
$\overline{c}$	1	Scalar of vacancy cost function
ϕ	3.45	Curvature of vacancy cost function
η	0.50	Elasticity of matches w.r.t. vacancies
ϵ	0.50	Innovation elasticity
π	0.23	Instantaneous profit per product line
N	1.95	Total mass of inventors
β	0.50	Bargaining power
		B. Estimated Offline
$\overline{\alpha}$	0.6141	Scalar of knowledge diffusion function
$\overline{\mu}$	0.0245	Scalar of internal R&D function
σ	0.1980	Std. of internal R&D shocks
		C. Internally Estimated
$\overline{q}$	0.0206	Matching efficiency
λ	1.0449	Step size on quality ladder

4.1 Methodology

First, we make assumptions regarding the functional forms. The vacancy cost function is $c(v) = \frac{\overline{c}}{\phi+1} v^{\phi+1}$, as in Kaas and Kircher (2015). The matching function is Cobb–Douglas: A vacancy meets an inventor at rate $q(\theta) = \overline{q}\theta^{\eta-1}$. The internal R&D function is $\mu(x) = \bar{\mu}x^{\epsilon}$, as in Lentz and Mortensen (2008). Finally, we assume that the knowledge diffusion function is given by $\alpha(x, x') = \overline{\alpha}\frac{x}{x'}$. Note that $\frac{x}{x'}$ represents the ratio of the inventor productivity between the firm from which inventors are poached and the firm poaching the inventors. Table 4 summarizes all the assumed functional forms. These assumptions about the functional form leave 13 parameters to be determined. We estimate these parameters through the following three steps.

Externally Set or Normalized

As summarized in Table 5, the eight parameters are set to standard values or normalized. The discount rate, $\rho = 0.004$, reflects a 5% annual real interest rate. Since we cannot identify the scalar of vacancy cost function $\overline{c}$ and the matching efficiency $\overline{q}$ separately, we normalize $\overline{c}$. The vacancy cost elasticity, $\phi = 3.45$, is based on estimates from Bilal et al. (2022). The elasticity of matches with respect to vacancies, $\eta = 0.50$, is based on standard values found in the literature[21]. The innovation elasticity, $\epsilon = 0.50$, also follows the standard values in the literature (e.g., Acemoglu et al. (2018)). The instantaneous profit per product line, $\pi = 0.23$, is set so that the markup equals 1.3, following recent research on markup estimates (see Cavenaile et al. (2023)). The total mass of inventors, $N = 1.95$, is chosen to align with an average productivity of German inventors, measured by the count of three-year forward citations[22]. When we calibrate the model, the value of β is set to 0.5. In the subsequent analysis, we examine in detail how changes in the value of β affect the allocation.

Estimated Offline

The parameters of the knowledge diffusion function $\overline{\alpha}$ and the internal R&D function $\left(\overline{\mu}, \sigma\right)$ are obtained by estimating the law of motion for inventor productivity. By subtracting (15) from (16), we obtain the law of motion for inventor productivity:

$$
\begin{aligned}
d \log x^{-1}(x,t) = & \frac{q(\theta(t))v(x,t)}{x} \int_0^\infty \mathbb{1}_P(x,x',t)\alpha(x,x') f_x(x',t)dx'dt \\
& + \left(\mu(x) - \delta(t) - \frac{\sigma^2}{2}\right)dt - d\log n(x,t) + \sigma dW(t).
\end{aligned}
$$

Given the functional forms and innovation elasticity of $\epsilon = 0.5$, we estimate the following regression equation:

$$
\begin{aligned}
\text{Inventor productivity growth rate}_{e,t+1} = \ & \overline{\alpha} \sum_{e'} \text{Inventor inflow rate}_{e' \to e,t} \\
& \times \text{Relative inventor productivity}_{e' \to e,t} \\
& + \overline{\mu}\Delta\left(\text{Inventor productivity}_{e,t}\right)^{-0.5} \\
& + \text{Controls}_{e,t} + \varepsilon_{e,t}
\end{aligned}
$$

where $\text{Controls}_{e,t}$ include net inventor inflow rate, firm fixed effect, and year fixed effect, and $\varepsilon_{e,t}$ has variance $\sigma^2\Delta$. The productivity of inventors is measured by the 3-year forward citations of their patents. The dependent variable is the growth rate of the average inventor productivity for inventors who belong to the establishment e in both t and $t+1$[23]. Inventor inflow $\text{rate}_{e' \to e,t}$ is the number of inventors newly hired by establishment e from

[21] See Petrongolo and Pissarides (2001) for a review.

[22] In our model, the total knowledge capital is 1. Therefore, the average inventor productivity in the model is $1/N$. Since the average number of 3-year forward citations for German inventors is 0.512, we set the total mass of inventors such that $1/N = 0.512$.

[23] To exclude the direct impact of the productivity of newly arrived inventors in the establishment, these inventors are excluded when calculating the average inventor productivity in period $t+1$.

establishment e' at time t, divided by the number of employed inventors in establishment e at time t. Relative inventor productivity$_{e' \to e,t}$ is the average inventor productivity in establishment e at time t, divided by that in establishment e' at time t. The standard deviation of internal R&D shocks σ is estimated using the value of the root mean square error obtained from the regression analysis. We set $\Delta = 12$ because the regression analysis uses yearly panel data, while the model's parameters are calibrated so that one unit of time corresponds to one month. The detailed explanation of the regression analysis and the reporting of its results have been relegated to the Appendix.

Internally Estimated

The remaining two parameters, matching efficiency $\overline{q}$ and innovation step size λ, are estimated to match the moments in the data with those in the model. Specifically, we first estimate $\overline{q}$ to match the 13% annual job-to-job transition rate of German inventors in 2001. As shown in Definition 1, the economic growth rate and λ do not affect the competitive equilibrium. Therefore, after estimating $\overline{q}$, we estimate λ to match the 2% annual economic growth rate.

5 Quantitative Exercises

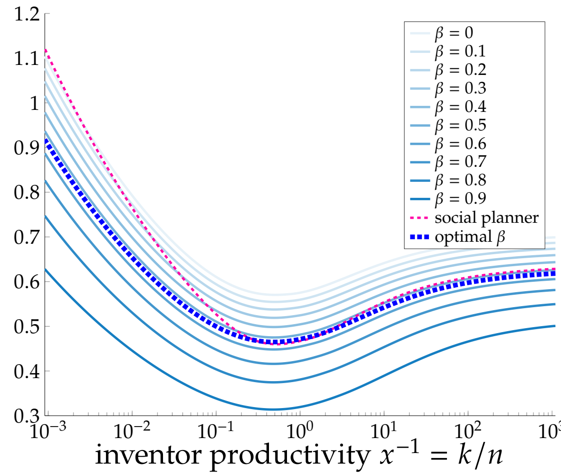
In this section, we begin by examining the allocation outcomes under different values of bargaining power, as well as the socially optimal allocation. We then analyze the effects of a ban on non-compete clauses over the transition. Finally, we assess the impact of the observed decline in inventor mobility in Germany on knowledge diffusion, internal R&D activities, and economic growth.

5.1 Comparative Statics for Bargaining Power and Social Optimum

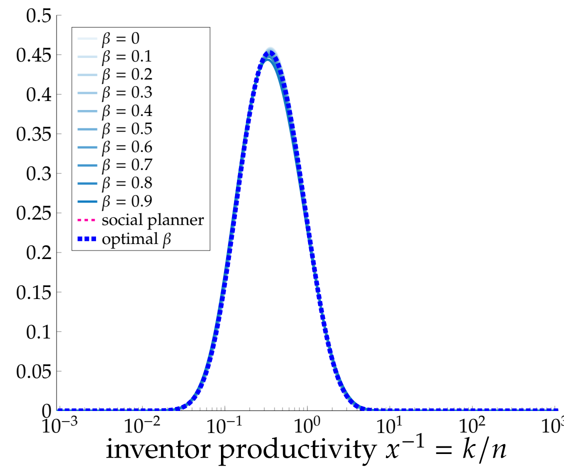
We first analyze the characteristics of the steady state of this economy. To examine the impact of the strength of non-compete clauses on the economy, we conduct comparative statics on the bargaining power β. Additionally, we compare these allocations with the socially optimal allocation.

As shown in Figure 1a, a firm's vacancy posting policy is U-shaped with respect to inventor productivity. Posting more vacancies allows firms to hire more inventors, and at the same time, they acquire more knowledge capital through knowledge diffusion. Therefore, a firm's vacancy posting policy depends on both the marginal value of inventors and the marginal value of knowledge capital for the firm. When the number of inventors n

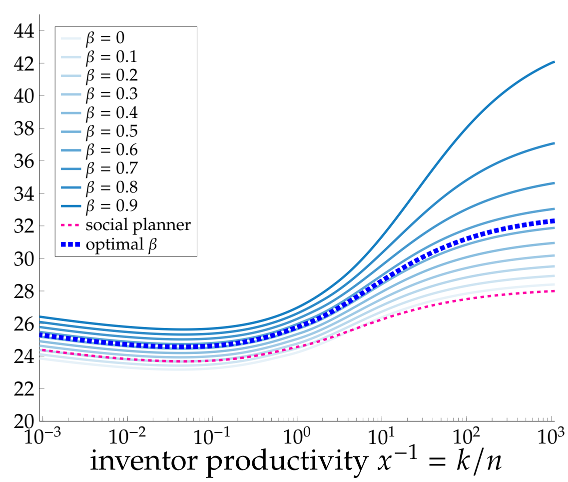
Figure 1: Equilibrium values under various β and social optimum



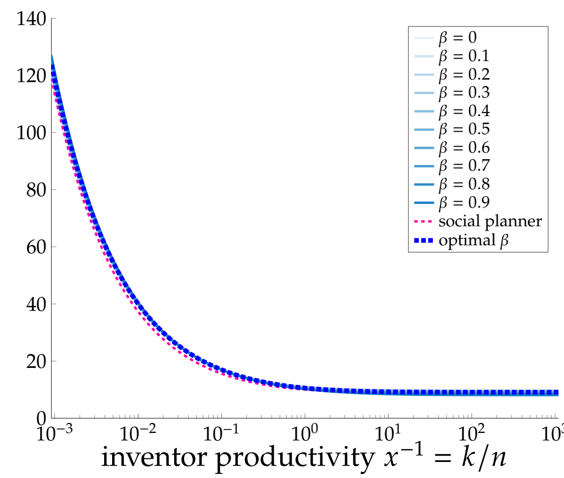
(a) Vacancy posting policy $v(x)$



(b) Inventor-weighted dist. $f_x(x)$

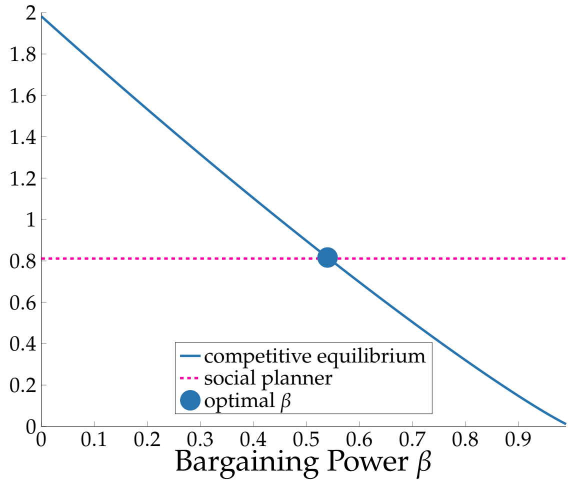


(c) $\Omega_n(k,n) = S_x(x)$

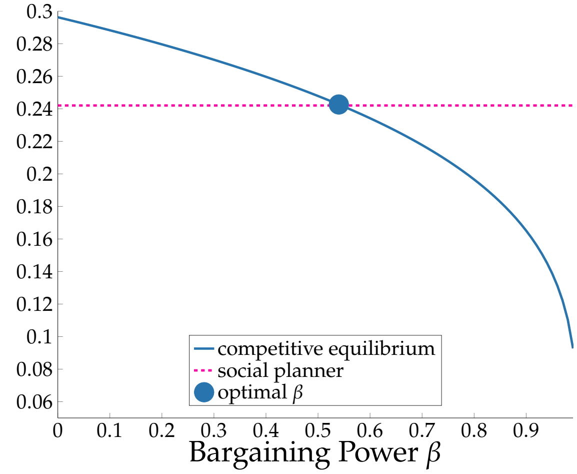


(d) $\Omega_k(k,n) = S(x) - xS_x(x)$

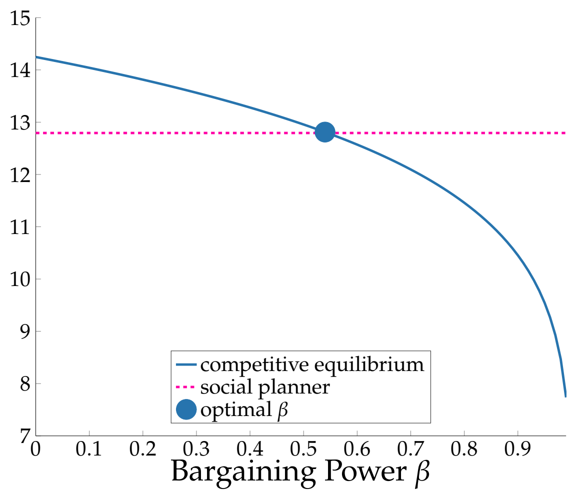
Figure 2: Aggregate variables under various β and social optimum



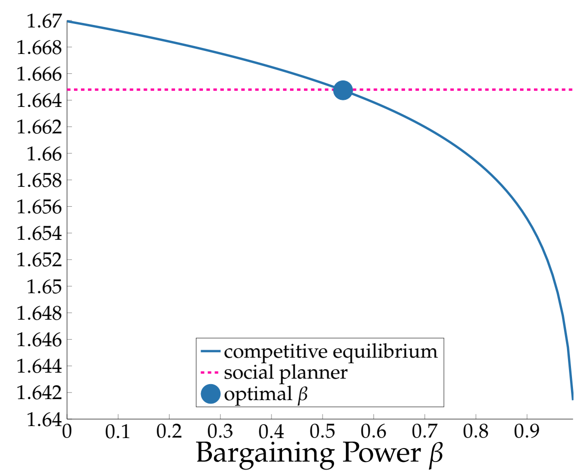
(a) Vacancy posting costs / Output (%)



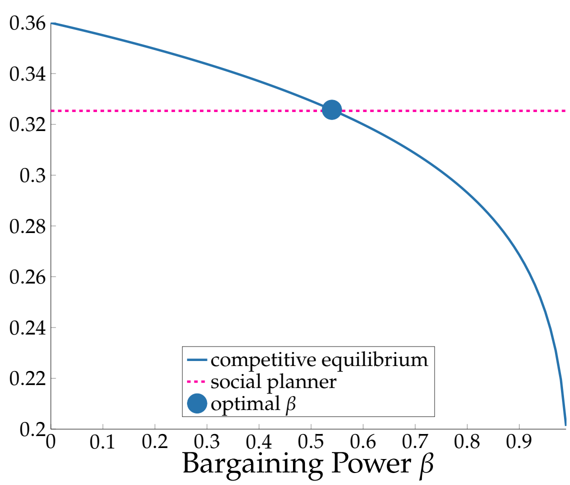
(b) Labor market tightness θ



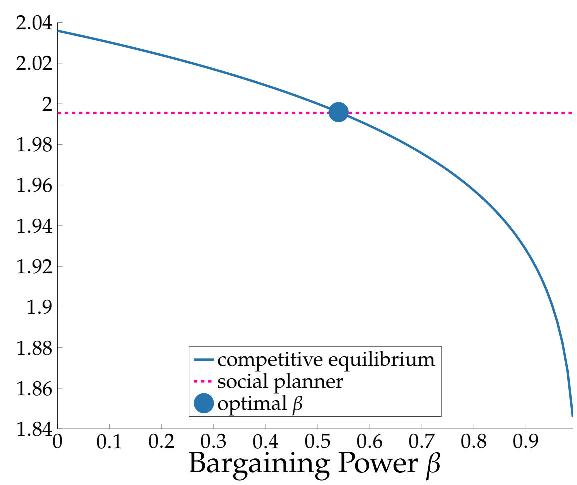
(c) Job-to-job transition rate (%)



(d) Internal R&D growth (%)



(e) Knowledge diffusion growth (%)



(f) Economic growth rate (%)

is small relative to knowledge capital k (i.e., inventor productivity $x^{-1} = k/n$ is high), the marginal value of inventors is large (Figure 1c). Conversely, when knowledge capital k is small relative to the number of inventors n (i.e., inventor productivity $x^{-1} = k/n$ is low), the marginal value of knowledge capital is large (Figure 1d). As a result, a firm's vacancy posting policy becomes U-shaped with respect to inventor productivity.

Figure 1a also shows that the quantity of vacancy postings decreases as β increases. In Section 3, we discussed several effects of the strength of non-compete clauses on firms' vacancy posting decisions. In our model, the strengthening of non-compete clauses primarily results in (i) a smaller share of the match surplus being obtained by the poaching firm. Our results indicate that this effect always outweighs two other effects: (ii) a reduction in vacancy postings by other firms, and (iii) an increase in the match surplus received by the firm whose inventor is poached.

Figure 2 illustrates the values of aggregate variables. As β increases, each firm's vacancy posting decreases, leading to a reduction in the share of total vacancy posting costs in total output (Figure 2a). As a result, labor market tightness decreases (Figure 2b), and the job-to-job transition rate also declines (Figure 2c) as β increases.

Figure 2d illustrates the economic growth driven by internal R&D. In our model, since the supply of inventors is inelastic, growth from internal R&D is determined by the allocative efficiency of inventors. Growth driven by inventor R&D is maximized when inventor productivity $x^{-1} = k/n$ is equal across all firms. However, the misallocation of inventors is inevitable in equilibrium due to (i) frictions in the labor market for inventors, (ii) idiosyncratic shocks, and (iii) the concavity of the internal R&D technology. A reduction in β, which lowers inventor mobility, worsens the allocation of inventors and reduces growth from internal R&D.

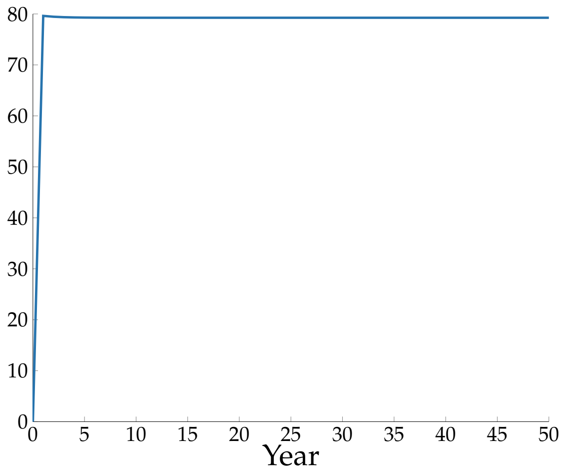
Figure 2e illustrates the economic growth driven by knowledge diffusion. Growth from knowledge diffusion accounts for 16.6% of total growth, while the remaining 83.4% is driven by internal R&D. However, the sensitivity of knowledge diffusion to changes in inventor mobility is much greater than that of internal R&D. Therefore, the decline in economic growth due to an increase in β (Figure 2f) is primarily driven by the reduction in knowledge diffusion growth.

In Figure 1, we also plot each variable under the social optimum and the optimal β[24]. The vacancy posting policy under the optimal β closely aligns with the socially optimal vacancy posting policy, especially in regions where the density of inventor productivity is high. As a result, as shown in Figure 2, the aggregate variables under the optimal β achieve values that are very close to the social optimum.

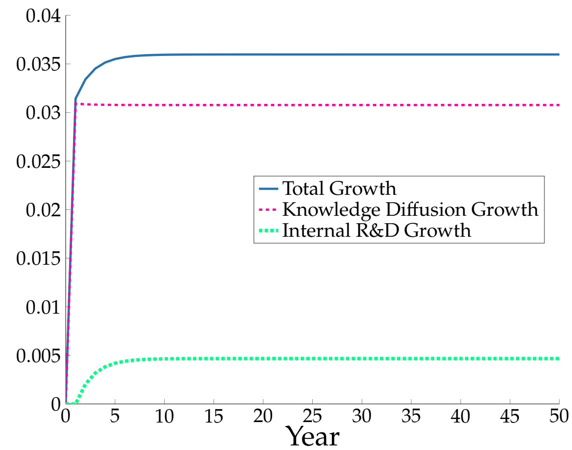
[24]We discuss the way to find the optimal β in Section 5.2.

5.2 Ban on Non-compete Clauses and Welfare Implication

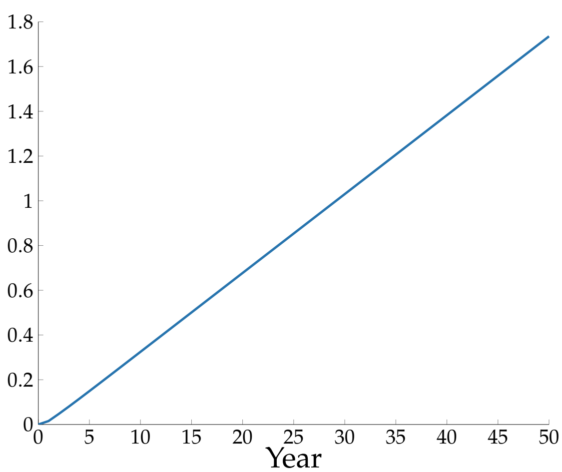
Figure 3: Transition dynamics after the ban on non-compete agreements



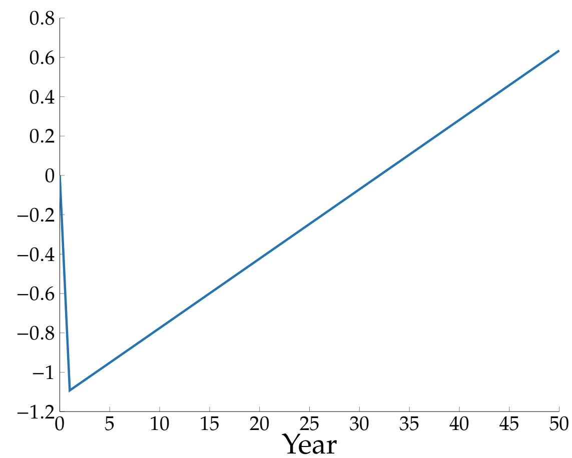
(a) Vacancy cost relative to initial BGP (%)



(b) Growth rates relative to initial BGP (%)



(c) $\log Y$ relative to initial BGP (%)

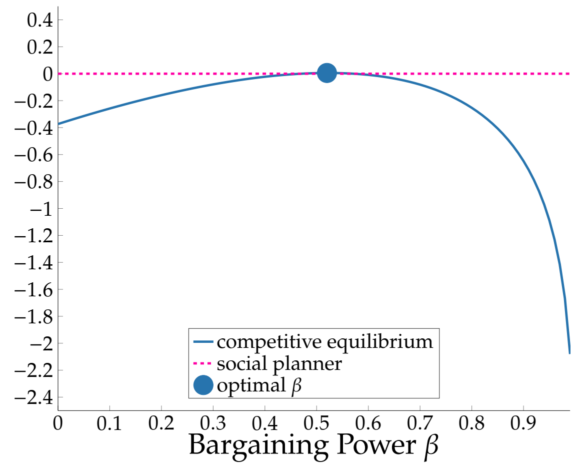


(d) $\log C$ relative to initial BGP (%)

In recent years, several countries have implemented or are planning to implement prohibitions or restrictions on non-compete clauses. For example, in April 2024, the Federal Trade Commission (FTC) banned all non-compete agreements in the US. In May 2023, the UK Government also announced plans to limit non-compete clauses to a maximum duration of three months.

To analyze the impact of banning non-compete clauses on economic growth and social welfare, we examine the transition dynamics of our economy. We assume that before $t = 0$, the economy is in a steady state on the BGP under the baseline value of bargaining

Figure 4: CE welfare change from social optimum (%)



power ($\beta = 0.5$). At $t = 0$, the bargaining power permanently changes to $\beta = 0$. Although agents do not anticipate this change before $t = 0$, agents have perfect foresight after $t = 0$. We interpret the economy with $\beta = 0$ as one where non-compete clauses are completely banned. This is because when $\beta = 0$, a firm matching an inventor can successfully poach by compensating the inventor for their marginal value at the firm where they are currently employed.

Figure 3 illustrates the transition dynamics of the economy after the policy change, compared with the BGP of the economy where $\beta = 0.5$. When β becomes 0, the match surplus received by firms that poach inventors increases, leading firms to post more vacancies. As a result, as shown in Figure 3a, the aggregate cost of posting vacancies, relative to output, increases by approximately 80%. As shown in Figure 3b, the increase in inventor mobility not only enhances knowledge diffusion but also improves the allocative efficiency of inventors, which in turn leads to higher internal R&D growth. Consequently, the total growth rate of the economy increases, leading to a rise in output, as depicted in Figure 3c. However, since the vacancy costs, which use final goods, increase, consumption temporarily decreases, as shown in Figure 3d.

In the economy after the policy change, the consumption-equivalent welfare decreases by 0.38% compared to the economy without the policy change. The reason why the prohibition of non-compete agreements does not have a large impact on welfare is that this policy has both positive effects (an increase in growth due to increased mobility of inventors) and negative effects (a decrease in consumption due to an increase in vacancy costs), which offset each other.

What is the optimal regulation of non-compete clauses, and to what extent does it alleviate inefficiencies in the economy? To answer these questions, we conduct the following

Figure 5: Inventor mobility decline and its impact on growth rates

(a) Job-to-job transition rate (%)

(b) CE welfare change (%)

exercise. Suppose the economy starts from the steady state of the social optimum. At $t = 0$, the economy is decentralized, and thereafter, agents have perfect foresight. Then, we compare social welfare along the transition dynamics of this decentralized equilibrium with social welfare along the BGP in which the economy continues to achieve the socially optimal allocation.

In Figure 4, we perform this exercise for different values of β, ranging from 0 to 0.99 in increments of 0.01. For each β, we calculate the difference in social welfare compared to the socially optimal BGP and plot the results. The optimal β in Figures 1 and 2 is the value of bargaining power ($\beta = 0.54$) that achieves the highest social welfare in this exercise. Figure 4 shows that under the optimal β, the social welfare achieved is almost the same as that in the social optimum. In other words, the optimal regulation of non-compete clauses achieves nearly the maximum attainable social welfare, given the production, innovation, and matching technology.

5.3 Inventor Mobility Decline and Growth

In this section, we analyze the impact of declining inventor mobility on economic growth. As shown in Figure 5a, data from Germany indicate that the job-to-job transition rate for inventors decreased from 13.0% in 2001 to 5.1% in 2014. An analysis by Akcigit and Goldschlag (2023a), using data on American inventors, shows that the hiring rate for inventors dropped from 7% in 2000, the first observable year, to 3.5% in 2016, the last observable year[25]. While our data is at the establishment level and their data is at the firm level—resulting in a consistently higher job-to-job transition rate in our data—both datasets indicate that inventor mobility halved over a similar period.

To analyze the impact of the observed decline in inventor mobility on economic growth and welfare, we calibrate the path of matching efficiency to match the dynamics of job-to-job transition rates of inventors in Germany with the model counterpart. Following Akcigit and Ates (2023), we assume that the path of matching efficiency takes the following simple functional form:

$$\overline{q}_t = \overline{q}_0 + \frac{\exp\left(-(t/T)\nu\right) - 1}{\exp\left(-\nu\right) - 1}\left(\overline{q}_T - \overline{q}_0\right) \quad \text{for all } t \in [0, T].$$

During the period from $t = 0$ to $t = T$, the matching efficiency $\overline{q}_t$ transitions smoothly from $\overline{q}_0$ to $\overline{q}_T$. The parameter ν adjusts the speed at which $\overline{q}_t$ changes. Until $t = 0$, agents do not anticipate the changes in $\overline{q}_t$, and after $t = 0$, agents have perfect foresight. After $t = T$, the matching efficiency is fixed at $\overline{q}_t = \overline{q}_T$, but this does not necessarily imply that the economy reaches a new BGP at $t = T$. The economy continues to converge to the new BGP even after $t = T$.

We calibrate the path of matching efficiency as follows. In our exercise, we set $t = 0$ to correspond to the year 2001 and $t = T$ to the year 2014. For $\overline{q}_0$, we use the value obtained from the steady-state calibration in Section 4. In Section 4, the calibration was conducted to match the job-to-job transition rate in 2001, which is the year corresponding to $t = 0$. The remaining parameters, ν and $\overline{q}_T$, are calibrated to match the job-to-job transition rates in 2007, which is the intermediate year between $t = 0$ and $t = T$, and in 2014, the year corresponding to $t = T$. Along with the targeted job-to-job transition rate of German inventors, Figure 5a plots the path of the job-to-job transition rate in the calibrated model.

Figure 5b illustrates the impact of declining inventor mobility on the economic growth rate. A reduction in inventor mobility decreases the growth rate by lowering both internal R&D and knowledge diffusion. As discussed earlier, the decline in inventor mobility exacerbates the misallocation of inventors, resulting in slower internal R&D growth. Furthermore, diminished inventor mobility weakens knowledge diffusion between firms. Consequently, in the long run, growth from internal R&D declines by 0.04 percentage points, while growth from knowledge diffusion drops by 0.20 percentage points, leading to a total reduction of 0.24 percentage points in the economic growth rate. Given that the economic growth rate in advanced economies is around 2%, the observed decline in inventor mobility has a significant impact, reducing the growth rate by over 10%. Moreover,

[25]See Figure 11 in their paper

consumption-equivalent welfare decreases by 3.2% compared to social welfare along the initial BGP, where matching efficiency remains constant.

6 Conclusion

In this paper, we examine how inventor mobility between firms governs knowledge diffusion, long-run growth, and welfare. Using matched German patent and employer-employee records, we document that job-to-job moves are associated with stronger knowledge diffusion across firms.

Motivated by these patterns, we build an endogenous economic growth model in which inventors advance technology both through in-house R&D and by transporting knowledge when they switch employers. Calibrated to German data, the framework replicates the mobility dynamics in the data and lets us assess labor-market policies that shape knowledge diffusion.

Counterfactual exercises deliver three key insights. First, banning non-compete clauses reduces consumption-equivalent welfare by 0.38%. Second, optimally regulating non-compete enforcement yields allocations and welfare close to the social planner's solution. Third, the observed decline in inventor mobility since the early 2000s lowers growth from internal R&D by 0.04 percentage points and growth from knowledge diffusion by 0.20 percentage points, cutting overall growth by 0.24 percentage points and reducing welfare by 3.2% relative to the initial balanced growth path.

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Appendix

Appendix A Empirical Appendix

A.1 Data

Our analyses utilize two administrative data sets, "Linked Inventor Biography Data 1980-2014" (INV-BIO) and "Sample of Integrated Labor Market Biographies" (Stichprobe der Integrierten Arbeitsmarktbiografien - SIAB). Both data sets are constructed by the Institute for Employment Research (IAB).

The SIAB data is a 2% random sample from the Integrated Employment Biographies (IEB). The IEB combines data from five different sources, each of which may contain information from various administrative procedures. It comprises all individuals in Germany who hold at least one of the following employment statuses: employment subject to social security, marginal part-time employment, receipt of benefits according to the German Social Code III or II, official registration as a job seeker at the German Federal Employment Agency, and (planned) participation in programs of active labor market policies (Dauth and Eppelsheimer 2020 for more detail).

The patent information contained in the INV-BIO dataset is sourced from register data recorded in PATSTAT, which includes bibliographical, procedural, and legal status information on patent applications handled by the European Patent Office. Additionally, data from DPMAregister, the online patent register of the German Patent and Trademark Office, is incorporated to enhance the PATSTAT data extract. The DPMAregister provides exclusive records of national patent applications that are not transferred to the European Patent Office or filed under the PCT (Patent Cooperation Treaty) route. As a result, the INV-BIO dataset comprises inventors who are listed on patent filings at the European Patent Office (EPO) between 1999 and 2011 and have been successfully linked with IEB (Dorner et al. 2018 for more detail). Table A.1 shows the summary statistics for INV-BIO and SIAB, respectively.

A.2 Robustness Check of Empirical Analyses

Table A.2 shows the transition matrix of inventor flows with wage increases, suggesting many flows from more productive establishments to less productive ones, even in this sample.

Instead of the probit model in Section 2.2, we estimate the following equation to control

Table A.6: **Table A.1:** Summary Statistics

(A) INV-BIO

Establishment level variables	Mean	S.D.	N of establishments (thous.)
N of inventors (n_{et})	4.9	18.5	119
N of employees	688.9	2150.6	119
Mean daily wage, Euro	121.6	55.5	119
N of three-year forward citations for patents (three-year backward average, z_{et})	11.3	69.2	119
Share of inventors moving from higher productivity establishments (H-Share$_{et}$), %	61.2	49.5	119
Total inventor inflows	1.67	5.30	119

(B) SIAB

Worker level variables	Mean	S.D.	N of workers (thus.)
Dummy for moving to less productive establishments (D_{it})			
based on establishment size	0.50	-	4,669
based on mean wage	0.52	-	4,583
Dummy for the identified inventors (I_{it})	0.10	-	5,691
Daily Wage, Euro	44.4	42.1	5,691
Age	33.7	12.9	5,691
Share of Women, %	47.3	-	5,691

for fixed effects,

$$D_{it} = \beta_0 + \beta_1 I_{it} + \beta_2 X_{it} + \alpha_e + \alpha_t + \varepsilon_{it} \tag{20}$$

Definitions of variables are the same as in Section 2.2. Table A.3 shows that inventors are more likely to move to less productive establishments conditional on fixed effects.

Table A.7: **Table A.2:** Transition Probabilities of Inventor Flows with Wage Increases

(A) Rank by Citation/Inventor

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	2.7	0.2	0.3	0.4	4.1
Origin	50-60	2.1	0.2	0.2	0.3	3.1
establishment	60-70	2.3	0.2	0.3	0.4	3.6
rank	70-80	2.7	0.2	0.3	0.4	3.6
	80-100	19.0	1.8	2.1	3.1	45.9

(B) Rank by Establishment Size

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	3.8	1.2	0.9	0.9	6.6
Origin	50-60	0.4	0.8	0.9	0.5	2.1
establishment	60-70	0.4	0.2	1.1	1.2	2.8
rank	70-80	0.5	0.3	0.4	1.9	4.9
	80-100	3.5	1.5	2.1	3.1	58.2

(C) Rank by Mean Wage

Share of flows (%)		Destination establishment rank				
		≤ 50%	50-60	60-70	70-80	80-100
	≤ 50%	4.4	1.5	1.3	1.4	4.4
Origin	50-60	0.9	1.3	1.6	1.1	2.5
establishment	60-70	0.8	0.9	2.6	2.9	4.1
rank	70-80	0.7	0.6	1.6	4.8	7.4
	80-100	2.0	2.6	2.8	4.9	42.4

Table A.8: **Table A.3:** Estimation Result for Inventor Flows (Linear Model)

	D_{it}			
	Whole sample		Sample with wage ↑	
I_{it}	.008***	.010**	.009***	.014***
	(.003)	(.004)	(.003)	(.004)
Control	√	√	√	√
Fixed Effects	√	√	√	√
Measure for D_{it}	Size	Mean wage	Size	Mean wage
N	2,938,537	2,959,368	1,609,460	1,617,613
Adj. R^2	.25	.22	.21	.20

Appendix B Theoretical Appendix

B.1 Household's Problem

The representative household has the preference

$$\int_0^\infty e^{-\rho t} \log C(t) dt.$$

and faces the budget constraint

$$\frac{d}{dt}A(t) = r(t)A(t) + w(t)L(t) + I(t) - P(t)C(t)$$

where $I(t)$ is the aggregate income of inventors. Therefore, the solution of the household problem maximizes the Hamiltonian:

$$\mathcal{H}^H(t, A(t), C(t), \lambda(t)) = \log C(t) + \lambda(t)\left[r(t)A(t) + w(t)L(t) + I(t) - P(t)C(t)\right],$$

where $\lambda(t)$ is the costate variable. We obtain the first order conditions

$$\frac{1}{C(t)} = \lambda(t)P(t) \tag{21}$$

$$\rho\lambda(t) - \dot{\lambda}(t) = \lambda(t)r(t), \tag{22}$$

and the transversality condition

$$\lim_{t\to\infty}\left[\exp\left(-\rho t\right)\lambda(t)A(t)\right] = 0.$$

Now focus on a balanced growth path (BGP). The normalization $P(t)Y(t) = 1$ keeps nominal output constant, and on a BGP the consumption-output ratio is time-invariant, so $P(t)C(t)$ is constant as well. Equation (21) then implies that $\lambda(t)$ is constant. Substituting this into (22) yields the Euler equation $r = \rho$. Along a BGP all real quantities are stationary, so $C(t)$, $w(t)L(t)$, $I(t)$, and $r(t)$ are constant. The household budget constraint therefore implies $\dot{A}(t) = 0$ and $A(t) = \bar{A}$. The transversality condition reduces to $\lim_{t\to\infty}\left[\exp\left(-\rho t\right)\bar{A}\right] = 0$, which holds whenever $\rho > 0$.

B.2 Final Good Producer's Problem

Given prices, the competitive final good producers maximize the profit

$$\Pi(t) = P(t)Y(t) - \int_0^1 p(\omega,t)l(\omega,t)d\omega,$$

subject to the technology

$$\log Y(t) = \int_0^1 \log\left(z(\omega,t)l(\omega,t)\right)d\omega = \int_0^1 \log z(\omega,t)d\omega + \int_0^1 \log l(\omega,t)d\omega.$$

Therefore, the FOCs are given by

$$\begin{aligned} P(t)\frac{\partial Y(t)}{\partial l(\omega,t)} &= p(\omega,t) \\ P(t)\left(\frac{\partial \log Y(t)}{\partial Y(t)}\right)^{-1}\frac{\partial \log Y(t)}{\partial l(\omega,t)} &= p(\omega,t) \\ P(t)Y(t) &= p(\omega,t)l(\omega,t). \end{aligned}$$

The choice of the numeraire $P(t)Y(t) = 1$ implies

$$p(\omega,t)l(\omega,t) = 1$$

for all $\omega \in [0,1]$ and $t > 0$.

B.3 Social Planner's Problem

In this appendix, we set the current value Hamiltonian of the social planner's problem and show the relationship between the variables in Definition 2 and the costate variables.

Define the state variables $\mathbf{x}(t)$ and control variables $\mathbf{y}(t)$ as:

$$\begin{aligned} \mathbf{x}(t) &= \left[\log Y(t), \left\{f(x,t)\right\}_{x\in(0,\infty)}\right] \\ \mathbf{y}(t) &= \left[\delta(t), \theta(t), \{v(x,t)\}_{x\in(0,\infty)}, \{\mathbb{1}_P(x,x',t)\}_{x,x'\in(0,\infty)}\right] \end{aligned}$$

Here $f(x,t)$ is the cross-sectional density over inventor types, and $f_x(x',t)$ and $f_v(x',t)$ denote the densities over incumbent partner types and vacancy types implied by the matching technology introduced above. The planner chooses $\mathbf{x} \equiv \{\mathbf{x}(t)\}_{t\in(0,\infty)}$ and $\mathbf{y} \equiv \{\mathbf{y}(t)\}_{t\in(0,\infty)}$ to maximize the social welfare (17) subject to the constraints (4) , (13), (7), (12), the initial log output $\log Y(0) = \log L$, and the initial density $f(x,0)$ that satisfies $1 = \int_0^\infty f(x,0)dx$ and $N = \int_0^\infty x f(x,0)dx$.

Let

$$\boldsymbol{\lambda}_{\mathbf{x}}(t) = \left[\lambda_Y(t), \left\{\lambda_f(x,t)\right\}_{x\in(0,\infty)}\right]$$

denote the costate variables associated with the constraint (4) and (12), and

$$\boldsymbol{\lambda}_{\mathbf{y}}(t) = [\lambda_\delta(t), \lambda_\theta(t)]$$

denote the costate variables associated with constraint (13) and (7). Then, we set the current value Hamiltonian of the social planner's problem:

$$\begin{aligned}
&\mathcal{H}^P\left(\mathbf{x}(t), \mathbf{y}(t), \boldsymbol{\lambda}_{\mathbf{x}}(t), \boldsymbol{\lambda}_{\mathbf{y}}(t)\right) \\
&= \ln Y(t) + \ln\left\{1 - \int_0^\infty c\left(v(x,t)\right) f(x,t)dx\right\} \\
&\quad + \lambda_Y(t)\,\delta(t)\ln\lambda \\
&\quad + \lambda_\delta(t)\left[-\delta(t) + \int_0^\infty \left\{\mu(x) + \frac{q(\theta(t))v(x,t)}{x}\int_0^\infty \mathbb{1}_P(x,x',t)\alpha(x,x')f_x(x',t)dx'\right\} f(x,t)dx\right] \\
&\quad + \lambda_\theta(t)\left\{\theta(t) - \int_0^\infty v(x,t)f(x,t)dx\right\} \\
&\quad + \int_0^\infty \lambda_f(x,t)\left[\begin{array}{c}
\left\{\frac{q(\theta(t))v(x,t)}{x}\int_0^\infty \mathbb{1}_P(x,x',t)\alpha(x,x')f_x(x',t)dx' + \mu(x) - \delta(t)\right\} f(x,t) \\
-\frac{\partial}{\partial x}\left[\left\{\begin{array}{c}
\frac{q(\theta(t))v(x,t)}{x}\int_0^\infty \mathbb{1}_P(x,x',t)\left\{1-\alpha(x,x')\right\}f_x(x',t)dx' \\
-\theta(t)\,q(\theta(t))\int_0^\infty \mathbb{1}_P(x',x,t)f_v(x',t)dx' \\
-\mu(x) + \delta(t)
\end{array}\right\} x f(x,t)\right] \\
+\frac{\partial^2}{\partial x^2}\left[\frac{\sigma^2}{2}x^2 f(x,t)\right]
\end{array}\right] dx
\end{aligned}$$

The optimality conditions consist of

$$0 = \frac{\partial}{\partial \mathbf{y}(t)}\mathcal{H}^P\left(\mathbf{x}(t), \mathbf{y}(t), \boldsymbol{\lambda}_{\mathbf{x}}(t), \boldsymbol{\lambda}_{\mathbf{y}}(t)\right) \tag{23}$$

$$\rho\boldsymbol{\lambda}_{\mathbf{x}}(t) - \dot{\boldsymbol{\lambda}}_{\mathbf{x}}(t) = \frac{\partial}{\partial \mathbf{x}(t)}\mathcal{H}^P\left(\mathbf{x}(t), \mathbf{y}(t), \boldsymbol{\lambda}_{\mathbf{x}}(t), \boldsymbol{\lambda}_{\mathbf{y}}(t)\right) \tag{24}$$

$$0 = \frac{\partial}{\partial \boldsymbol{\lambda}_{\mathbf{y}}(t)}\mathcal{H}^P\left(\mathbf{x}(t), \mathbf{y}(t), \boldsymbol{\lambda}_{\mathbf{x}}(t), \boldsymbol{\lambda}_{\mathbf{y}}(t)\right) \tag{25}$$

$$\dot{\mathbf{x}}(t) = \frac{\partial}{\partial \boldsymbol{\lambda}_{\mathbf{x}}(t)}\mathcal{H}^P\left(\mathbf{x}(t), \mathbf{y}(t), \boldsymbol{\lambda}_{\mathbf{x}}(t), \boldsymbol{\lambda}_{\mathbf{y}}(t)\right) \tag{26}$$

and the transversality conditions

$$\lim_{t\to\infty}\left[\exp\left(-\rho t\right)\boldsymbol{\lambda}_{\mathbf{x}}\left(t\right)\mathbf{x}\left(t\right)\right]=0.$$

We can confirm that, when $\rho > 0$, the transversality condition is satisfied.

Define

$$\begin{aligned} S\left(x,t\right) &\equiv \lambda_f\left(x,t\right)+\lambda_\delta\left(t\right) \\ \pi\left(t\right) &\equiv \left(\rho+\delta\left(t\right)\right)\lambda_\delta\left(t\right) \end{aligned}$$

Then, rearranging the optimality conditions (23)–(26) gives the conditions in Definition 2.

Appendix C Quantitative Appendix

C.1 The Estimation of the Law of Motion for Inventors' Productivity

	Inventor productivity growth rate			
	Unweighted		Weighted by # of inventors	
$\sum_{e'}\left[\text{Inventor inflow rate}_{e'\to e,t}\times \text{Relative inventor productivity}_{e'\to e,t}\right]$	.352***	.389***	.456***	.614***
	(.042)	(.053)	(.066)	(.088)
$\left(\text{Inventor productivity}_{e,t}\right)^{-\frac{1}{2}}$	.296***	.442***	.168***	.293***
	(.007)	(.010)	(.012)	(.023)
Net inventor inflow rate	✓	✓	✓	✓
Year fixed effect	✓	✓	✓	✓
Firm fixed effect		✓		✓
Root mean square error	0.955	0.886	0.697	0.686
N	12,596	11,458	12,178	11,037

Notes: SEs are reported in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

C.2 HJB Equation

Note that

$$\begin{aligned} &\int\left[S_x(x)+\alpha\left(x,x'\right)x^{-1}\left\{S(x)-xS_x(x)\right\}-S_x(x')\right]^+dF_x(x') \\ &=\int \mathbb{1}_P(x,x')\left[\alpha\left(x,x'\right)x^{-1}S(x)+\left\{1-\alpha\left(x,x'\right)\right\}S_x(x)-S_x(x')\right]dF_x(x') \end{aligned}$$

$$
\begin{aligned}
&= \int \mathbb{1}_P(x,x')\alpha(x,x')\,x^{-1}dF_x(x')S(x) \\
&\quad + \int \mathbb{1}_P(x,x')\left\{1-\alpha(x,x')\right\}dF_x(x')S_x(x) \\
&\quad - \int \mathbb{1}_P(x,x')S_x(x')dF_x(x')
\end{aligned}
$$

and

$$
\begin{aligned}
&\int \left[S_x(x') + \alpha(x',x)\,x'^{-1}\left\{S(x') - x'S_x(x')\right\} - S_x(x)\right]^+ dF_v(x') \\
&= \int \mathbb{1}_P(x',x)\left[\alpha(x',x)\,x'^{-1}S(x') + \left\{1-\alpha(x',x)\right\}S_x(x') - S_x(x)\right]dF_v(x') \\
&= -\int \mathbb{1}_P(x',x)dF_v(x')S_x(x) \\
&\quad + \int \mathbb{1}_P(x',x)\left\{\alpha(x',x)\,x'^{-1}S(x') + \left\{1-\alpha(x',x)\right\}S_x(x')\right\}dF_v(x')
\end{aligned}
$$

Then, the HJB equation is rewritten as

$$
\begin{aligned}
\rho S(x) =& \pi - c\left(v\left(x\right)\right) \\
&+ \left\{\mu\left(x\right) - \delta\right\}\left\{S(x) - xS_x(x)\right\} + \frac{\sigma^2}{2}x^2 S_{xx}(x) \\
&+ \left\{\left(1-\beta\right)\frac{q(\theta)v\left(x\right)}{x}\int \mathbb{1}_P(x,x')\alpha\left(x,x'\right)dF_x(x')\right\}S(x) \\
&+ \left\{\begin{array}{c} \left(1-\beta\right)\frac{q(\theta)v(x)}{x}\int \mathbb{1}_P(x,x')\left\{1-\alpha\left(x,x'\right)\right\}dF_x(x') \\ -\beta\theta q(\theta)\int \mathbb{1}_P(x',x)dF_v(x') \end{array}\right\}xS_x(x) \\
&- \left(1-\beta\right)q(\theta)v\left(x\right)\int \mathbb{1}_P(x,x')S_x(x')dF_x(x') \\
&+ \beta\theta q(\theta)x\int \mathbb{1}_P(x',x)\left\{\alpha\left(x',x\right)x^{-1}S(x') + \left\{1-\alpha\left(x',x\right)\right\}S_x(x')\right\}dF_v(x')
\end{aligned}
$$

We solve the HJB equation using an implicit method. Let Δ denote the step-size and τ the iteration of the algorithm. Then, given $S^{\tau-1}(x)$, the implicit method gives an update

$$
\begin{aligned}
\rho S^\tau(x) - \frac{1}{\Delta}\left[S^{\tau-1}(x) - S^\tau(x)\right] =& \pi - c\left(v(x)\right) \\
&+ \left\{\mu(x) - \delta\right\}\left\{S^\tau(x) - xS^\tau_x(x)\right\} + \frac{\sigma^2}{2}x^2 S^\tau_{xx}(x) \\
&+ (1-\beta)\frac{q(\theta)v(x)}{x}\left[\int \mathbb{1}_P(x,x')\alpha(x,x')\,dF_x(x')\right]S^\tau(x)
\end{aligned}
$$

$$
\begin{aligned}
&+\Bigg[(1-\beta)\frac{q(\theta)v(x)}{x}\int \mathbb{1}_P(x,x')\left\{1-\alpha(x,x')\right\}\, dF_x(x') \\
&\qquad -\beta\theta q(\theta)\int \mathbb{1}_P(x',x)\, dF_v(x')\Bigg] x S_x^{\tau}(x) \\
&-(1-\beta)q(\theta)v(x)\int \mathbb{1}_P(x,x')S_x^{\tau-1}(x')\, dF_x(x') \\
&+\beta\theta q(\theta)x\int \mathbb{1}_P(x',x)\Big[\alpha(x',x)x^{-1}S^{\tau-1}(x') \\
&\qquad +\left\{1-\alpha(x',x)\right\} S_x^{\tau-1}(x')\Big]\, dF_v(x')
\end{aligned}
$$

$$
\begin{aligned}
&\left[\frac{1}{\Delta}+\rho-\left\{\mu(x)-\delta\right\}-(1-\beta)\frac{q(\theta)v(x)}{x}\int \mathbb{1}_P(x,x')\alpha(x,x')\, dF_x(x')\right] S^{\tau}(x) \\
&\quad -\Bigg[-\mu(x)+\delta+(1-\beta)\frac{q(\theta)v(x)}{x}\int \mathbb{1}_P(x,x')\left\{1-\alpha(x,x')\right\}\, dF_x(x') \\
&\qquad -\beta\theta q(\theta)\int \mathbb{1}_P(x',x)\, dF_v(x')\Bigg] x S_x^{\tau}(x)-\frac{\sigma^2}{2}x^2 S_{xx}^{\tau}(x) \\
&=\frac{1}{\Delta}S^{\tau-1}(x)+\pi-c\left(v(x)\right) \\
&\quad -(1-\beta)q(\theta)v(x)\int \mathbb{1}_P(x,x')S_x^{\tau-1}(x')\, dF_x(x') \\
&\quad +\beta\theta q(\theta)x\int \mathbb{1}_P(x',x)\Big[\alpha(x',x)x^{-1}S^{\tau-1}(x') \\
&\qquad +\left\{1-\alpha(x',x)\right\} S_x^{\tau-1}(x')\Big]\, dF_v(x')
\end{aligned}
$$

Let $\tilde{x}=\log x$. Then,

$$
\begin{aligned}
&\left[\frac{1}{\Delta}+\rho-\left\{\tilde{\mu}(\tilde{x})-\delta\right\}-\left(1-\beta\right)\frac{q(\theta)\tilde{v}(\tilde{x})}{\exp(\tilde{x})}\int \mathbb{1}_P(\tilde{x},\tilde{x}')\tilde{\alpha}(\tilde{x},\tilde{x}')\, d\tilde{F}_{\tilde{x}}(\tilde{x}')\right]\tilde{S}^{\tau}(\tilde{x}) \\
&-\left[\begin{array}{c} -\left\{\tilde{\mu}(\tilde{x})-\delta+\frac{\sigma^2}{2}\right\} \\ +\left(1-\beta\right)\frac{q(\theta)\tilde{v}(\tilde{x})}{\exp(\tilde{x})}\int \mathbb{1}_P(\tilde{x},\tilde{x}')\left\{1-\tilde{\alpha}(\tilde{x},\tilde{x}')\right\}\, d\tilde{F}_{\tilde{x}}(\tilde{x}')-\beta\theta q(\theta)\int \mathbb{1}_P(\tilde{x}',\tilde{x})\, d\tilde{F}_{\tilde{v}}(\tilde{x}') \end{array}\right]\tilde{S}_{\tilde{x}}^{\tau}(\tilde{x}) \\
&-\frac{\sigma^2}{2}\tilde{S}_{\tilde{x}\tilde{x}}^{\tau}(\tilde{x}) \\
&\quad =\frac{1}{\Delta}\tilde{S}^{\tau-1}(\tilde{x})+\pi-c\left(\tilde{v}(\tilde{x})\right)
\end{aligned}
$$

$$- \left(1-\beta\right) q(\theta)\tilde{v}\left(\tilde{x}\right) \int \mathbb{1}_P(\tilde{x},\tilde{x}') \exp\left(-\tilde{x}'\right) \tilde{S}_{\tilde{x}}^{\tau-1}(\tilde{x}')\, d\tilde{F}_{\tilde{x}}(\tilde{x}')$$
$$+ \beta\theta q(\theta) \exp\left(\tilde{x}\right) \int \mathbb{1}_P(\tilde{x}',\tilde{x}) \exp\left(-\tilde{x}'\right) \left\{\tilde{\alpha}\left(\tilde{x}',\tilde{x}\right) \tilde{S}^{\tau-1}(\tilde{x}') + \left\{1-\tilde{\alpha}\left(\tilde{x}',\tilde{x}\right)\right\} \tilde{S}_{\tilde{x}}^{\tau-1}(\tilde{x}')\right\} d\tilde{F}_{\tilde{v}}(\tilde{x}') \tag{27}$$

We now discretize $\tilde{x}$ on an evenly spaced N_x grid. Stack these according to:

$$\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_{N_x} \end{pmatrix}$$

Then, (27) can be rewritten in vector form as follows:

$$A_1 S^\tau - A_2 S_x^\tau - \frac{\sigma^2}{2} S_{xx}^\tau = \frac{1}{\Delta} S^{\tau-1} + A_0$$

where the element of N_x vector S^τ consists of $\tilde{S}^\tau(\tilde{x})$, the element of N_x vector S_x^τ consists of $\tilde{S}_x^\tau(\tilde{x})$, the element of N_x vector S_{xx}^τ consists of $\tilde{S}_{xx}^\tau(\tilde{x})$, and the ith element of the N_x-vector A_0 is

$$\pi - c\left(\tilde{v}\left(\tilde{x}\right)\right) - \left(1-\beta\right) q(\theta)\tilde{v}\left(\tilde{x}\right) \int \mathbb{1}_P(\tilde{x},\tilde{x}') \exp\left(-\tilde{x}'\right) \tilde{S}_{\tilde{x}}^{\tau-1}(\tilde{x}')\, d\tilde{F}_{\tilde{x}}(\tilde{x}')$$
$$+ \beta\theta q(\theta) \exp\left(\tilde{x}\right) \int \mathbb{1}_P(\tilde{x}',\tilde{x}) \exp\left(-\tilde{x}'\right) \left\{\tilde{\alpha}\left(\tilde{x}',\tilde{x}\right) \tilde{S}^{\tau-1}(\tilde{x}') + \left\{1-\tilde{\alpha}\left(\tilde{x}',\tilde{x}\right)\right\} \tilde{S}_{\tilde{x}}^{\tau-1}(\tilde{x}')\right\} d\tilde{F}_{\tilde{v}}(\tilde{x}'),$$

Let a_1 be the N_x-vector with elements

$$\frac{1}{\Delta} + \rho - \left\{\tilde{\mu}\left(\tilde{x}\right) - \delta\right\} - \left(1-\beta\right) \frac{q(\theta)\tilde{v}\left(\tilde{x}\right)}{\exp\left(\tilde{x}\right)} \int \mathbb{1}_P(\tilde{x},\tilde{x}')\tilde{\alpha}\left(\tilde{x},\tilde{x}'\right)\, d\tilde{F}_{\tilde{x}}(\tilde{x}'),$$

and let a_2 be the N_x-vector with elements

$$-\left\{\tilde{\mu}\left(\tilde{x}\right) - \delta + \frac{\sigma^2}{2}\right\} + \left(1-\beta\right) \frac{q(\theta)\tilde{v}\left(\tilde{x}\right)}{\exp\left(\tilde{x}\right)} \int \mathbb{1}_P(\tilde{x},\tilde{x}') \left\{1-\tilde{\alpha}\left(\tilde{x},\tilde{x}'\right)\right\}\, d\tilde{F}_{\tilde{x}}(\tilde{x}') - \beta\theta q(\theta) \int \mathbb{1}_P(\tilde{x}',\tilde{x})\, d\tilde{F}_{\tilde{v}}(\tilde{x}').$$

Define the diagonal matrices $A_1 = \text{diag}(a_1)$ and $A_2 = \text{diag}(a_2)$.

Let D_x be the $N_x \times N_x$ matrix that, when pre-multiplying S^τ gives an approximation of S_x^τ, and let D_{xx} be the $N_x \times N_x$ matrix that, when pre-multiplying S^τ gives an approximation of S_{xx}^{τ}:

$$S_x^{\tau} = D_x S^{\tau}$$
$$S_{xx}^{\tau} = D_{xx} S^{\tau}$$

To compute the derivative matrices D_x, we follow an upwind scheme. That is, we use a forward approximation when the drift of the state variable is positive, and a backward approximation when the drift of the state is negative.

The implicit method works by updating S^{τ} through the following equation:

$$S^{\tau} = \left\{ A_1 - A_2 D_x - \frac{\sigma^2}{2} D_{xx} \right\}^{-1} \left\{ \frac{1}{\Delta} S^{\tau-1} + A_0 \right\}$$

C.3 KF Equation

Analogous to the HJB equation, given $f^{\tau-1}(x)$, the implicit method provides the update:

$$\begin{aligned}
&\frac{1}{\Delta}\left[\tilde{f}^{\tau-1}(\tilde{x}) - \tilde{f}^{\tau}(\tilde{x})\right] \\
&\quad = \left\{ q(\theta)\tilde{v}(\tilde{x}) \int \mathbb{1}_P(\tilde{x}, \tilde{x}')\tilde{\alpha}(\tilde{x}, \tilde{x}')\, d\tilde{F}_{\tilde{x}}^{\tau-1}(\tilde{x}') + \tilde{\mu}(\tilde{x}) - \delta \right\} \tilde{f}^{\tau}(\tilde{x}) \\
&\qquad - \frac{\partial}{\partial \tilde{x}}\left[\left\{ \begin{array}{c} \frac{q(\theta)\tilde{v}(\tilde{x})}{\exp(\tilde{x})} \int \mathbb{1}_P(\tilde{x}, \tilde{x}')\{1 - \tilde{\alpha}(\tilde{x}, \tilde{x}')\}\, d\tilde{F}_{\tilde{x}}^{\tau-1}(\tilde{x}') - \theta q(\theta) \int \mathbb{1}_P(\tilde{x}', \tilde{x})\, d\tilde{F}_{\tilde{v}}^{\tau-1}(\tilde{x}') \\ - \left\{\tilde{\mu}(\tilde{x}) - \delta + \frac{\sigma^2}{2}\right\} \end{array} \right\} \tilde{f}^{\tau}(\tilde{x})\right] \\
&\qquad + \frac{\sigma^2}{2}\frac{\partial^2}{\partial \tilde{x}^2}\tilde{f}^{\tau}(\tilde{x})
\end{aligned}$$

This can be rewritten in vector form as follows:

$$\begin{aligned}
-\frac{1}{\Delta}\left(f^{\tau} - f^{\tau-1}\right) &= B_1 f^{\tau} - D_x B_2 f^{\tau} + \frac{\sigma^2}{2} D_{xx} f^{\tau} \\
\frac{1}{\Delta} f^{\tau-1} &= \left\{ \frac{1}{\Delta} + B_1 - D_x B_2 + \frac{\sigma^2}{2} D_{xx} \right\} f^{\tau} \\
f^{\tau} &= \left\{ \frac{1}{\Delta} + B_1 - D_x B_2 + \frac{\sigma^2}{2} D_{xx} \right\}^{-1} \frac{1}{\Delta} f^{\tau-1},
\end{aligned}$$

where the element of N_x vector f^{τ} consists of $\tilde{f}^{\tau}(\tilde{x})$, the element of $N_x \times N_x$ diagonal matrix B_1 consists of

$$q(\theta)\tilde{v}(\tilde{x}) \int \mathbb{1}_P(\tilde{x}, \tilde{x}')\tilde{\alpha}(\tilde{x}, \tilde{x}')\, d\tilde{F}_{\tilde{x}}^{\tau-1}(\tilde{x}') + \tilde{\mu}(\tilde{x}) - \delta$$

and the element of N_x vector B_2 consists of

$$\frac{q(\theta)\tilde{v}(\tilde{x})}{\exp(\tilde{x})}\int \mathbb{1}_P(\tilde{x},\tilde{x}')\{1-\tilde{\alpha}(\tilde{x},\tilde{x}')\}\, d\tilde{F}_{\tilde{x}}^{\tau-1}(\tilde{x}') - \theta q(\theta)\int \mathbb{1}_P(\tilde{x}',\tilde{x})\, d\tilde{F}_{\tilde{v}}^{\tau-1}(\tilde{x}')$$
$$-\left\{\tilde{\mu}(\tilde{x}) - \delta + \frac{\sigma^2}{2}\right\}.$$

To construct the derivative matrices, we use a backward approximation when the drift of the state variable is positive, and a forward approximation when the drift of the state is negative.

C.4 Consumption-Equivalent Welfare Gains

Calculation of Consumption Path $\{C(t)\}_{t\geq 0}$

From the resource constraint (14),

$$\log C(t) = \log Y(t) + \log\left\{1 - \int_0^\infty c(v(x,t))\, dF(x,t)\right\}$$

Inserting (3), we obtain

$$\log C(t) = \log \lambda \int_0^t \delta(\tau)\, d\tau + \log L + \log\left\{1 - \int_0^\infty c(v(x,t))\, dF(x,t)\right\} \tag{28}$$

Therefore, we calculate the consumption path $\{C(t)\}_{t\geq 0}$ from the path of creative destruction rate $\{\delta(t)\}_{t\geq 0}$ and the current vacancy posting $v(x,t)$ and distribution $F(x,t)$ using (28).

Calculation of Welfare $V(\{C(t)\}_{t\geq 0})$

Assume that after period T, $C(t)$ grows at the economic growth rate in the final steady state $\overline{g}$. Then, we calculate $V(\{C(t)\}_{t\geq 0})$ numerically from $\{C(t)\}_{t\in[0,T]}$ and $\overline{g}$ as follows:

$$\begin{aligned}
V(\{C(t)\}_{t\geq 0}) &= \int_0^\infty e^{-\rho t}\log C(t)dt \\
&= \int_0^T e^{-\rho t}\log C(t)dt + \int_T^\infty e^{-\rho t}\log C(t)dt \\
&= \int_0^T e^{-\rho t}\log C(t)dt + \int_T^\infty e^{-\rho t}\log\left\{C(T)e^{(t-T)\overline{g}}\right\} dt
\end{aligned}$$

$$= \int_0^T e^{-\rho t} \log C(t) dt + \frac{1}{\rho} e^{-\rho T} \left\{ \log C(T) + \frac{\overline{g}}{\rho} \right\}$$

Calculation of Consumption-Equivalent Welfare Gains $\mathcal{L}$

Definition 3. Let $\{C(t)\}_{t\geq 0}$ denote the consumption path in the economy without policy change and $\{\widetilde{C}(t)\}_{t\geq 0}$ denote the consumption path in the economy with policy change. *Consumption-equivalent welfare gains from policy change* is the scalar $\mathcal{L}$ such that the consumer is indifferent between the consumption path $\{\mathcal{L} \times C(t)\}_{t\geq 0}$ and the consumption path $\{\widetilde{C}(t)\}_{t\geq 0}$.

Let $V(\{C(t)\}_{t\geq 0})$ denote the welfare:

$$V(\{C(t)\}_{t\geq 0}) \equiv \int_0^\infty e^{-\rho t} \log C(t) dt.$$

Then,

$$\begin{aligned} V(\mathcal{L} \times \{C(t)\}_{t\geq 0}) &= \int_0^\infty e^{-\rho t} \log(\mathcal{L} \times C(t)) dt \\ &= \frac{\log \mathcal{L}}{\rho} + \int_0^\infty e^{-\rho t} \log C(t) dt \\ &= \frac{\log \mathcal{L}}{\rho} + V(\{C(t)\}_{t\geq 0}) \end{aligned}$$

This implies

$$\frac{\log \mathcal{L}}{\rho} = V(\mathcal{L} \times \{C(t)\}_{t\geq 0}) - V(\{C(t)\}_{t\geq 0})$$

or equivalently,

$$\mathcal{L} = \exp\left[\rho \{V(\mathcal{L} \times \{C(t)\}_{t\geq 0}) - V(\{C(t)\}_{t\geq 0})\}\right]$$

Therefore, the consumption-equivalent welfare gain from policy change is calculated as

$$\mathcal{L} = \exp\left[\rho \left\{V\left(\left\{\tilde{C}(t)\right\}_{t\geq 0}\right) - V(\{C(t)\}_{t\geq 0})\right\}\right]$$