Ownership Structure and Economic Growth

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Abstract

This paper examines how the rise of common ownership affects economic growth and social welfare. We develop an endogenous growth model that incorporates three inter-firm networks: ownership, product-market rivalry, and innovation. In the model, a large number of oligopolistic firms make forward-looking R&D investment decisions, internalizing externalities on commonly owned firms arising from product-market competition and technological spillovers. We estimate the model using data on over 700 publicly traded U.S. firms with patents. Our counterfactual analysis shows that the observed increase in common ownership between 1999 and 2017 reduced the annual growth rate by 0.12 percentage points and social welfare by 0.6%. This finding suggests that, under common ownership, the internalization of the negative externality from innovation that reduces competitors' market shares dominates the internalization of the positive externality associated with technological spillovers.

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1 Introduction

Over the past few decades, corporate ownership in the United States has become increasingly concentrated among a small set of institutional investors, a trend often referred to as the rise of common ownership (Backus et al., 2021). Today, three institutional investors—BlackRock, Vanguard, and State Street—collectively control roughly 30 percent of shareholder votes at S&P 500 firms[1]. When common owners hold stakes in competing firms, managers seeking to maximize investor returns may partially internalize the externalities their actions impose on rivals' firm values (Rotemberg, 1984; Azar and Ribeiro, 2021). As a result, researchers and policymakers have become increasingly concerned about the implications of such concentrated ownership for product-market competition and firms' R&D activities (Ederer and Tecu, 2025)[2].

Firms' R&D activities generate two inter-firm externalities: business-stealing and technology spillovers (Bloom et al., 2013). Business-stealing causes negative externalities that lower competitors' market shares, whereas technology spillovers raise productivity for firms that rely on similar technologies. Micro-level evidence shows that common ownership induces firms to partially internalize both forces (Anton et al., 2024; Kini et al., 2024). Ederer and Pellegrino (2024) analyze how common ownership affects the allocative efficiency of product markets through its impact on markups and competition, but we still lack a framework that quantifies how common ownership, via its effects on firms' R&D incentives, shapes aggregate productivity growth. In particular, it remains unclear whether the rise of common ownership leads firms to internalize positive or negative innovation externalities more strongly and, in turn, whether common ownership ultimately promotes or hinders economic growth and social welfare.

To address these questions, we develop an endogenous growth model featuring three inter-firm networks: product-market rivalry, innovation, and ownership. The product-market rivalry network governs the strength of business-stealing externalities, whereas the innovation network governs the strength of technology spillovers. The ownership network determines how strongly firms internalize the value of other firms held by common owners. Consequently, firms internalize these externalities more when the ownership network more closely overlaps with the product-market or innovation networks. We estimate

[1] Highly concentrated corporate ownership is not unique to the United States. For example, in South Korea the four largest conglomerates (the "chaebols") account for roughly 40% of the country's stock market capitalization. Likewise, in Japan during the first half of the 20th century, three "zaibatsu" controlled about 30% of the total assets of the top 100 manufacturing firms (Takeda, 2020).

[2] For example, the 2023 Merger Guidelines issued by the U.S. Department of Justice and the Federal Trade Commission express concern that common ownership may discourage the development of new product features that would reduce the value of jointly owned firms.

these networks using data on more than 700 publicly listed U.S. patenting firms. We then solve for the equilibrium of the dynamic oligopoly model and assess how rising common ownership affects economic growth and social welfare.

Our model extends the static product-market competition framework of Pellegrino (2024) and Ederer and Pellegrino (2024). Pellegrino (2024) apply a widely used hedonic demand system from the empirical IO literature in a general-equilibrium environment and develop an associated estimation strategy. Building on this static block, we follow Hopenhayn and Okumura (2024) in embedding firms' forward-looking R&D decisions and technology spillovers. Firms accumulate knowledge capital through their own R&D investments, while technology spillovers from other firms augment the knowledge capital. The resulting knowledge capital raises product quality and labor productivity, thereby linking firms' R&D incentives to aggregate productivity growth.

We incorporate the ownership network into the endogenous growth model of Hopenhayn and Okumura (2024). In our model, owners receive profits in proportion to their shares, and firms make decisions to maximize the value of their owners' portfolios, as in Rotemberg (1984) and Ederer and Pellegrino (2024). Thus, each firm maximizes a weighted average of its own value and the values of other commonly owned firms. Consequently, firms partially internalize both business-stealing effects on commonly owned firms producing similar products and technology spillover effects on commonly owned firms with similar technologies. The integration of these three networks allows us to quantify how observed changes in common ownership reshape firms' R&D incentives, aggregate productivity growth, and social welfare.

Our model retains tractability despite incorporating forward-looking and dynamic R&D decisions by a large number of oligopolistic firms. We achieve this tractability by formulating the firms' dynamic R&D problem as a linear-quadratic differential game. This approach facilitates the analysis of dynamic oligopolies, even when the model includes hundreds of U.S. publicly listed firms with patent holdings. Although we assume non-CES demand to retain the linear-quadratic structure, the model admits a balanced growth path equilibrium with an endogenously determined growth rate. Furthermore, the framework offers another computational advantage when calculating the expected social welfare: once the equilibrium is solved, we can calculate the expected social welfare with minimal additional computational effort for any initial distribution of knowledge capital.

We begin by examining the model's theoretical properties. In this setting, a matrix referred to as the "technology transition matrix," which captures technological spillovers and endogenous R&D strategies, plays a central role in characterizing the conditions for the existence of a balanced growth path, the economic growth rate, and the stationary distribution of knowledge capital. Specifically, the matrix's irreducibility is sufficient for a globally stable balanced growth path. Intuitively, irreducibility means that all firms are technologically connected. Along the balanced growth path, the growth rate equals the matrix's dominant eigenvalue, and the distribution of knowledge capital across firms is proportional to the associated eigenvector.

We identify the model parameters using data on U.S. publicly traded patenting firms, totaling more than 700 firms in each year from 1999 to 2017. Following Pellegrino (2024), we measure product-market rivalry between each pair of U.S. patenting firms with Hoberg and Phillips (2016)'s text-based product similarity. To construct the technological spillover network, we follow Bloom et al. (2013) and compute technological proximity from patent classification data. Corporate ownership is taken from the institutional ownership dataset of Backus et al. (2021) based on 13F filings. Combining these networks with Compustat financials, we also recover the model-implied distribution of firm-level knowledge capital.

We estimate the relationship between the magnitude of technological spillovers across firms and their observed technological similarity to discipline our model. The main empirical challenge is to distinguish common shocks to firms' knowledge capital from true spillover effects. For example, when new research opportunities arise in a given technological field, all firms in that field may expand R&D and accumulate knowledge capital, which could be mistaken for spillovers. Following Wilson (2009); Bloom et al. (2013); Lucking et al. (2019), we construct an instrument for knowledge capital based on changes in the user cost of R&D induced by state-level tax variation. This strategy provides exogenous variation in knowledge capital, enabling causal identification of technological spillover effects.

Our quantitative analysis evaluates how alternative ownership structures shape firms' R&D investment, economic growth, and social welfare in an environment where ownership influences firms' R&D choices. We compare the observed 2017 ownership network with a counterfactual network that rescales the 2017 weights to match the average level of common ownership observed in 1999. Moving from the 1999-scaled ownership network to the 2017 network lowers total R&D expenditure from about 39% of the planner's allocation to about 28%.[3] This change reduces the annual growth rate from 1.3% to 1.2% and lowers consumption-equivalent welfare from 94.9 to 94.4.

These aggregate effects of common ownership arise because common ownership reshapes strategic interactions among firms. Greater overlap in ownership induces firms to internalize both business-stealing and technology-spillover externalities, yet our estimates

[3] Consistent with our results, Bloom et al. (2013) estimate that the socially optimal level of R&D is about three times the observed level.

show that the internalization of business-stealing effects dominates, so common ownership depresses R&D investment and growth. Because decentralized R&D already falls short of the socially optimal level, this further reduction generated by expanding common ownership lowers social welfare.

We also use the quantitative model to study R&D subsidies. First, we analyze the optimal level of a uniform subsidy.[4] The optimal uniform subsidy is a 43% subsidy on R&D expenditure. At that rate, the expected growth rate rises to 1.8%—about 0.6 percentage points above the benchmark with no subsidy—and consumption-equivalent welfare increases by about 1.4%. These gains, however, remain far smaller than the roughly 6% welfare increase delivered by the optimal R&D allocation, underscoring that policies should exploit network heterogeneity to achieve large effects.

To guide such targeted interventions, we regress the ratio of social to private R&D value on firm characteristics. The estimates indicate that firms with high product-market centrality should receive smaller R&D subsidies because they impose large business-stealing losses on their rivals. By contrast, firms with high innovation centrality generate large technology spillovers and thus higher social returns on R&D, so these firms should receive larger subsidies. Firms with high ownership centrality have ownership structures that strongly overlap with those of other firms, so they already partially internalize inter-firm externalities. Consequently, once R&D subsidies address the business-stealing and technology-spillover distortions, policymakers should provide additional support to these firms to offset the remaining gap between private and social incentives.

This study makes several contributions to the literature. First, we build on extensive research on innovation and product-market competition (d'Aspremont and Jacquemin, 1988; Aghion et al., 2001, 2005; Acemoglu and Akcigit, 2012; Bloom et al., 2013; Peters, 2020; Akcigit and Ates, 2021, 2023; Liu et al., 2022; Cavenaile et al., 2023; De Ridder, 2024). Bloom et al. (2013) empirically document how product-market and technological proximity shape firms' R&D activity: higher product-market proximity strengthens business-stealing incentives, whereas greater technological proximity amplifies technology spillovers. Cavenaile et al. (2023) develop a Schumpeterian growth model with oligopolistic competition among a small number of firms (at most four firms) within each industry, so strategic interaction arises only within industries; as a result, the framework cannot analyze how cross-industry technology spillovers—such as those studied by Liu and Ma (2024)—are internalized under common ownership. However, models with dynamic oligopoly face a curse of dimensionality, with computational costs growing exponentially in the number of firms. To

[4] A uniform subsidy shifts all firms' R&D incentives without altering the static production strategy profile, so welfare can improve to the extent that the subsidy closes the gap with the planner's R&D allocation.

address this challenge, Hopenhayn and Okumura (2024) embed a linear–quadratic network game in a Schumpeterian growth model, allowing the framework to capture externalities both within and across industries. Relative to existing endogenous growth models, this approach directly links firm- and firm-pair-level data to the model, enabling us to leverage newly available microdata and computational power. In this paper, we further extend the framework to incorporate ownership structure networks and study their implications for economic growth.

Our research contributes to the growing literature on common ownership and innovation (Gutierrez and Philippon, 2017; He and Huang, 2017; Lopez and Vives, 2019; Anton et al., 2023, 2024; Kini et al., 2024). Lopez and Vives (2019) analyze how common ownership affects innovation in a static symmetric-oligopoly setting. Anton et al. (2024) show that common ownership negatively affects innovation when firms are close in product markets but positively affects innovation when they are technologically proximate. They interpret this pattern as evidence of stronger business-stealing forces at high product-market proximity, stronger technology spillovers at high technological proximity, and partial internalization of these externalities under common ownership. Kini et al. (2024) employ a difference-in-differences design using mergers among financial institutions as an exogenous shock to common ownership and analyze impacts on investment and new product development. They find that common ownership raises these outcomes when technology spillovers are high. Our study differs from the literature on common ownership and innovation in two respects. First, we develop a dynamic general-equilibrium framework in which the economic growth rate is determined endogenously and the economy evolves along a (stochastic) balanced growth path. Second, we propose an empirical strategy to estimate heterogeneous inter-firm networks from data, enabling quantitative analysis of macroeconomic questions.

The remainder of the paper proceeds as follows. Section 2 constructs an endogenous growth model incorporating common ownership. Section 3 describes the data and the estimation method for the model. Section 4 presents baseline model characteristics and counterfactual results. Section 5 examines alternative corporate governance models. Section 6 concludes.

2 Model

We develop an endogenous economic growth model that incorporates three inter-firm networks: ownership structure, product market competition, and technology spillovers.

2.1 Environment

There are n oligopolistic firms, indexed by $i \in \{1, 2, \ldots, n\}$, and they determine production and dynamic R&D investment under strategic interaction through the networks. Time is infinite and continuous, $t \in [0, \infty)$.

2.1.1 Demand

We introduce the Generalized Hedonic Linear demand system, developed by Pellegrino (2024), to model product market rivalry among a large number of oligopolists. We assume that products have two types of characteristics. The first type of characteristics is a common characteristic shared by all products and indexed by $k \in \{1, 2, \ldots, m\}$. The second type of characteristics is an idiosyncratic characteristic unique to each product and shares the same index i as the product it corresponds to. The scalar $\psi_{ki} \geq 0$ denotes the quantity of the common characteristic k provided by product i. Each product i is described by an m-dimensional column vector with nonnegative entries $\boldsymbol{\psi}_i$:

$$\boldsymbol{\psi}_i = \begin{bmatrix} \psi_{1i} & \psi_{2i} & \ldots & \psi_{mi} \end{bmatrix}^\top .$$

The Euclidean length of the common-characteristics vector for each product is normalized to 1 without loss of generality:

$$\sum_{k=1}^{m} \psi_{ki}^2 = 1 \quad \forall i \in \{1, 2, \ldots, n\}.$$

We stack the vectors $\boldsymbol{\psi}_i$ across products and define the $m \times n$ matrix $\boldsymbol{\Psi}$ (columns are products, rows are common characteristics):

$$\boldsymbol{\Psi} \equiv \begin{bmatrix} \boldsymbol{\psi}_1 & \boldsymbol{\psi}_2 & \cdots & \boldsymbol{\psi}_n \end{bmatrix} \equiv \begin{bmatrix} \psi_{11} & \psi_{12} & \cdots & \psi_{1n} \\ \psi_{21} & \psi_{22} & \cdots & \psi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{m1} & \psi_{m2} & \cdots & \psi_{mn} \end{bmatrix} .$$

We aggregate the characteristics of all products to obtain the production function of the final-goods producer. Let $q_{i,t}$ denote the quantity of variety i produced by firm i and sold to the final-goods producer at time t, and define the n-dimensional vector $\boldsymbol{q}_t$:

$$\boldsymbol{q}_t \equiv \begin{bmatrix} q_{1,t} & q_{2,t} & \cdots & q_{n,t} \end{bmatrix}^\top .$$

Let $y^C_{k,t}$ be the total units of common characteristic k aggregated over all products:

$$y^C_{k,t} = \sum_{i=1}^{n} \psi_{ki} q_{i,t}.$$

Therefore, the matrix $\mathbf{\Psi}$ transforms units of goods into units of common characteristics:

$$\boldsymbol{y}^C_t = \mathbf{\Psi} \boldsymbol{q}_t. \tag{1}$$

Let $y^I_{i,t}$ be the number of units of idiosyncratic characteristic i, and assume that each unit of good i provides one unit of its corresponding idiosyncratic characteristic:

$$\boldsymbol{y}^I_t = \boldsymbol{q}_t. \tag{2}$$

A representative final-goods producer aggregates common and idiosyncratic characteristics using a linear-quadratic aggregator:

$$Y_t = \alpha \sum_{k=1}^{m} \left(b^C_{k,t} y^C_{k,t} - \frac{1}{2} \left(y^C_{k,t} \right)^2 \right) + (1-\alpha) \sum_{i=1}^{n} \left(b^I_{i,t} y^I_{i,t} - \frac{1}{2} \left(y^I_{i,t} \right)^2 \right), \tag{3}$$

where $b^C_{k,t} > 0$ and $b^I_{i,t} > 0$ are characteristic-specific quality, and $\alpha \in [0,1]$ is the weight that is assigned to common characteristics. Inserting equations (1) and (2) into equation (3), we obtain a vector form of aggregate output represented by the quantity vector:

$$Y_t = \boldsymbol{q}_t^\top \boldsymbol{b}_t - \frac{1}{2} \boldsymbol{q}_t^\top \mathbf{\Sigma} \boldsymbol{q}_t, \tag{4}$$

where we define

$$\boldsymbol{b}^C_t \equiv (b^C_{1,t}, \ldots, b^C_{m,t})^\top, \quad \boldsymbol{b}^I_t \equiv (b^I_{1,t}, \ldots, b^I_{n,t})^\top,$$

and set

$$\boldsymbol{b}_t \equiv \alpha \mathbf{\Psi}^\top \boldsymbol{b}^C_t + (1-\alpha) \boldsymbol{b}^I_t,$$

The remainder of the model works directly with this product-level quality vector. Thus, when firms choose $b_{i,t}$ below, the choice variable is the reduced-form quality that enters inverse demand, rather than the underlying characteristic-level qualities $\boldsymbol{b}^C_t$ and $\boldsymbol{b}^I_t$

separately.

$$\boldsymbol{\Sigma} \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix} \equiv \alpha \boldsymbol{\Psi}^\top \boldsymbol{\Psi} + (1-\alpha)\boldsymbol{I}, \tag{5}$$

and $\boldsymbol{I}$ is the $n \times n$ identity matrix.

$\boldsymbol{\Sigma}$ denotes the product-market rivalry matrix; each entry σ_{ij} measures substitutability between products i and j. As implied by Equation (4), larger σ_{ij} increases the penalty in final-good production when both q_i and q_j are high. By construction, $\sigma_{ii} = 1$ for all i. The matrix $\boldsymbol{\Sigma}$ is a weighted sum of product similarity $\boldsymbol{\Psi}^\top \boldsymbol{\Psi}$ and the identity. Elements of $\boldsymbol{\Psi}^\top \boldsymbol{\Psi}$ are given by $[\boldsymbol{\Psi}^\top \boldsymbol{\Psi}]_{ij} = \boldsymbol{\psi}_i^\top \boldsymbol{\psi}_j$, i.e., the cosine similarity of the common characteristics of products i and j (since $\boldsymbol{\psi}_i^\top \boldsymbol{\psi}_j$ is the cosine of the angle between $\boldsymbol{\psi}_i$ and $\boldsymbol{\psi}_j$ in $\mathbb{R}^m$). From Equation (5), for $i \neq j$ we have $\sigma_{ij} \propto \boldsymbol{\psi}_i^\top \boldsymbol{\psi}_j$, with the common-characteristics weight α scaling similarity into substitutability. As shown by Pellegrino (2024), this structure enables estimation of substitutability using product-similarity data.

2.1.2 Technology

Each firm has a linear production technology in labor:

$$q_{i,t} = a_{i,t} l_{i,t} \tag{6}$$

where $a_{i,t}$ is the labor productivity of firm i at time t and $l_{i,t}$ is the production labor employed by firm i at time t. Firms possess their own knowledge capital and utilize it to improve their labor productivity and product quality. As shown in Equation (4), $b_{i,t}$ represents the value, measured in final good units, generated by the first unit of product i. Therefore, $b_{i,t}$ can be interpreted as the quality of the product. Firms allocate their knowledge capital to improve labor productivity $a_{i,t}$ and product quality $b_{i,t}$:

$$\zeta a_{i,t} + b_{i,t} = z_{i,t} \tag{7}$$

where $\zeta > 0$ is a parameter that determines the transformation rate between improvements in labor productivity and product quality[5].

Now, we introduce the law of motion for knowledge capital, which increases through

[5] Assuming optimal allocation of knowledge capital between labor-productivity and quality improvements is necessary for a balanced-growth-path equilibrium. Similarly, Grossman et al. (2017) and Jones and Liu (2024) obtain a balanced growth path by assuming two productivity-enhancing technologies and allocating resources between them (capital and human capital; capital productivity and automation).

technology spillovers from technologically similar firms and the firm's own R&D investment. Let $\widetilde{\mathbf{\Omega}}$ denote the technological proximity between firms:

$$\widetilde{\mathbf{\Omega}} = \begin{bmatrix} \tilde{\omega}_{11} & \tilde{\omega}_{12} & \cdots & \tilde{\omega}_{1n} \\ \tilde{\omega}_{21} & \tilde{\omega}_{22} & \cdots & \tilde{\omega}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\omega}_{n1} & \tilde{\omega}_{n2} & \cdots & \tilde{\omega}_{nn} \end{bmatrix} \tag{8}$$

where each element $\tilde{\omega}_{ij} \geq 0$ denotes the technological proximity between firms i and j. We assume that all the diagonal elements of the matrix are equal to 0: $\tilde{\omega}_{ii} = 0$ for all i. Let $\boldsymbol{x}_t$ denote the vector of R&D efforts:

$$\boldsymbol{x}_t \equiv \begin{bmatrix} x_{1,t} & x_{2,t} & \dots & x_{n,t} \end{bmatrix}^\top \tag{9}$$

and let $\boldsymbol{W}_t$ denote the vector of independent standard Brownian motions:

$$\boldsymbol{W}_t \equiv \begin{bmatrix} W_{1,t} & W_{2,t} & \dots & W_{n,t} \end{bmatrix}^\top \tag{10}$$

We assume that firms' knowledge capital follows a multi-dimensional geometric Brownian motion with obsolescence at rate δ:

$$d\boldsymbol{z}_t = \left((\mathbf{\Omega} - \delta \boldsymbol{I})\, \boldsymbol{z}_t + \mu \boldsymbol{x}_t \right) dt + \gamma \text{diag}\left(\boldsymbol{z}_t\right) d\boldsymbol{W}_t \tag{11}$$

where $\mathbf{\Omega} = \beta \widetilde{\mathbf{\Omega}}$, and β, μ, γ, and δ are nonnegative scalars. We assume that the magnitude of technology spillovers between two firms is proportional to the technological proximity $\widetilde{\mathbf{\Omega}}$, and β is the parameter that converts technological proximity into the magnitude of technology spillovers. μ is a parameter that determines the efficiency of R&D and is common to all firms. γ is a parameter that determines the magnitude of shocks in the geometric Brownian motion, which is also common to all firms. δ captures the obsolescence/depreciation rate of knowledge capital. Here, diag($\boldsymbol{z}_t$) denotes the diagonal matrix with $\boldsymbol{z}_t$ on its diagonal.

2.1.3 Ownership Structure and Firm Objective Function

In our model, investors own multiple firms, and firms compete to maximize the weighted expected returns of their owners, with the weights determined by the proportion of shares. There are n_o investment funds indexed by o. Let $\tilde{V}^o$ represent the value of fund o and $\hat{V}^i$ represent the value of firm i. The value of fund $\tilde{V}^o$ is given as the weighted sum of the value of firms $\hat{V}^i$, where the weights correspond to the shareholding ratio of each firm held by the fund:

$$\tilde{V}^o \equiv \sum_{i=1}^{n} s_{io} \hat{V}^i \tag{12}$$

where s_{io} is the proportion of shares of firm i owned by fund o such that $\sum_{o=1}^{n_o} s_{io} = 1$. Following Rotemberg (1984) and Ederer and Pellegrino (2024), we assume that firm i maximizes the value V^i, which is the weighted sum of investment funds' values $\tilde{V}^o$, weighted by their respective ownership shares in firm i:

$$V^i \equiv \sum_{o=1}^{n_o} s_{io} \tilde{V}^o$$

By substituting the value of the fund Equation (12) into the above equation, we obtain the expression for the objective value of the firm, V^i, as a linear combination of the firm's value $\hat{V}^j$:

$$V^i = \sum_{j=1}^{n} \hat{V}^j \, \boldsymbol{s}_i^\top \boldsymbol{s}_j = \left(\boldsymbol{s}_i^\top \boldsymbol{s}_i\right) \sum_{j=1}^{n} \kappa_{ij} \hat{V}^j \propto \sum_{j} \kappa_{ij} \hat{V}^j,$$

where

$$\kappa_{ij} \equiv \frac{\boldsymbol{s}_i^\top \boldsymbol{s}_j}{\boldsymbol{s}_i^\top \boldsymbol{s}_i} \tag{13}$$

and

$$\boldsymbol{s}_i \equiv \begin{bmatrix} s_{i1} & s_{i2} & \ldots & s_{i,n_o} \end{bmatrix}^\top.$$

Because $\boldsymbol{s}_i^\top \boldsymbol{s}_i > 0$ is a constant for firm i, this scale factor does not affect the maximization problem. Note that firm i's value $\hat{V}^i$ is distinct from the objective value function V^i that the firm maximizes. The firm i's value $\hat{V}^i$ represents the expected discounted present value of the cash flows it generates. In contrast, the firm maximizes the weighted average of the value of its owners, which implies that the firm maximizes a linear combination of the values of firms, including those of other firms.

We define the common ownership weight matrix $\boldsymbol{K}$ as

$$\boldsymbol{K} \equiv \begin{bmatrix} \kappa_{11} & \kappa_{12} & \cdots & \kappa_{1n} \\ \kappa_{21} & \kappa_{22} & \cdots & \kappa_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \kappa_{n1} & \kappa_{n2} & \cdots & \kappa_{nn} \end{bmatrix}.$$

Note that, by construction, the diagonal elements $\kappa_{ii} = 1$ for all i. Common-ownership weight κ_{ij} is the relative weight on firm j's value in firm i's objective. The objective function nests standard profit maximization ($\kappa_{ij} = 0$ for $i \neq j$) and monopoly ($\kappa_{ij} = 1$ for $i \neq j$) as limit cases.

2.1.4 Resources and Household Preferences

We close the model with resource constraints and the representative household's preferences. Production labor is supplied inelastically (with $L > 0$ exogenous and constant):

$$L = \sum_i l_{i,t} \tag{14}$$

Final goods are allocated to consumption and R&D expenditures. Following the endogenous growth literature (e.g. Acemoglu et al. 2016), we assume an innovation elasticity of 1/2 at the firm level; with units normalized so that the cost coefficient equals one, total R&D expenditure is quadratic in effort:

$$C_t + \sum_i x_{i,t}^2 = Y_t \tag{15}$$

where C_t denotes final-good consumption at time t. The representative household is risk-neutral and discounts the future at rate $\rho > 0$; lifetime utility is

$$\boldsymbol{E}_0 \left[\int_0^\infty \exp\left(-\rho t\right) C_t \, dt \right] \tag{16}$$

2.2 Equilibrium

We focus on a Markov Perfect Equilibrium (MPE) and therefore separate the problem into static and dynamic components. First, we analyze the static problem of competition in the product market; then, we address the dynamic problem of R&D investments.

2.2.1 Final-Goods Producer

The final good is produced competitively. In each period, the representative final-goods producer chooses the vector of quantities to maximize profit:

$$\max_{\boldsymbol{q}_t} \; P_t Y_t - \boldsymbol{q}_t^\top \boldsymbol{p}_t$$

where $\boldsymbol{p}_t$ is the vector of product prices at time t and P_t is the final-good price. Profit maximization yields the linear inverse-demand function:

$$\frac{\boldsymbol{p}_t}{P_t} = \boldsymbol{b}_t - \boldsymbol{\Sigma}\boldsymbol{q}_t \tag{17}$$

2.2.2 Static Problem of Firms: Production Decision and Technology Choice

We assume that firms maximize the real returns of their owners to ensure independence of the allocation from the choice of numeraire (Azar and Vives, 2021). Thus, in the static problem, firms maximize the common-ownership-weighted sum of real profits. Each firm's real gross profit *before* subtracting R&D cost, $\pi_{i,t}$, is given by

$$\begin{aligned}\pi_{i,t} &= \frac{p_{i,t}}{P_t} q_{i,t} - \frac{w_t}{P_t} l_{i,t} \\ &= \left(b_{i,t} - \frac{1}{a_{i,t}} \frac{w_t}{P_t} - \sum_j \sigma_{ij} q_{j,t} \right) q_{i,t}\end{aligned} \tag{18}$$

In the first line on the right-hand side, the real labor cost is subtracted from real revenue. The second line is obtained by substituting the inverse demand in Equation (17) and the production function in Equation (6).

In the static problem, the objective of firm i is the common-ownership-weighted sum of real gross profits:

$$\sum_j \kappa_{ij} \pi_{j,t}$$

Each firm i simultaneously chooses quantity $q_{i,t}$ and the allocation of knowledge capital between labor productivity $a_{i,t}$ and product quality $b_{i,t}$, taking as given the output of other firms $\{q_{j,t}\}_{j \neq i}$, the real wage w_t/P_t, and its own knowledge capital $z_{i,t}$, subject to the knowledge-capital constraint Equation (7). The FOC with respect to $q_{i,t}$ gives

$$0 = b_{i,t} - \frac{1}{a_{i,t}} \frac{w_t}{P_t} - 2q_{i,t} - \sum_{j \neq i} \sigma_{ij} q_{j,t} - \sum_{j \neq i} \kappa_{ij} \sigma_{ji} q_{j,t} \tag{19}$$

From the optimality conditions for $a_{i,t}$ and $b_{i,t}$, we obtain

$$a_{i,t} = \sqrt{\frac{w_t}{\zeta P_t}} \tag{20}$$

$$b_{i,t} = z_{i,t} - \sqrt{\frac{\zeta w_t}{P_t}}. \tag{21}$$

Intuitively, firms invest more in labor productivity when labor costs w_t/P_t are higher. Inserting Equation (20) into the labor-market clearing condition Equation (14) yields

$$\sqrt{\zeta \frac{w_t}{P_t}} = \frac{\zeta}{L} \sum_i q_{i,t}. \tag{22}$$

Combining Equations (19) to (22) yields a linear relationship between quantities and knowledge capital:

$$\boldsymbol{q}_t = \boldsymbol{N} \boldsymbol{z}_t \tag{23}$$

where $\boldsymbol{N}$ is an $n \times n$ matrix:

$$\boldsymbol{N} \equiv \left(2\frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma} \right)^{-1},$$

$\boldsymbol{J}$ is an $n \times n$ matrix with all elements equal to one (see Appendix A.1 for the derivation). The first term inside the parentheses, $2\frac{\zeta}{L}\boldsymbol{J}$, captures labor costs; the second term $\boldsymbol{\Sigma}$ represents product-market rivalry; and the third term $\boldsymbol{K} \circ \boldsymbol{\Sigma}$ is the element-wise product of common-ownership weights and rivalry. Hence, in the static problem, ownership influences production only through its overlap with product-market rivalry.

Given the linear relationship between the quantity $\boldsymbol{q}_t$ and knowledge capital $\boldsymbol{z}_t$, we can express the firm's static objective in a quadratic form of the firm's productivity vector $\boldsymbol{z}_t$:

$$\sum_j \kappa_{ij} \pi_{j,t} = \boldsymbol{z}^\top \boldsymbol{Q}^i \boldsymbol{z}$$

where

$$\boldsymbol{Q}^i \equiv \sum_h \kappa_{ih} \sum_j \kappa_{hj} \sigma_{hj} \frac{\boldsymbol{N}_h^\top \boldsymbol{N}_j + \boldsymbol{N}_j^\top \boldsymbol{N}_h}{2}.$$

and $\boldsymbol{N}_i$ denotes the i-th row of $\boldsymbol{N}$ (see Appendix A.2 for the derivation).

2.2.3 Dynamic Problem of Firms: R&D Decision

Given a static production strategy profile, we consider a dynamic R&D game. In the dynamic game, given other players' strategies $\left\{x_{j,t}\right\}_{j \neq i,\, t \geq 0}$, firm i chooses R&D effort $\{x_{i,t}\}_{t \geq 0}$ to maximize the expected discounted present value of profits weighted by ownership weights:

$$\max_{\left\{x_{i,t}\right\}_{t \geq 0}} \quad V^i(z_0) \equiv \mathbb{E}_0 \left[\int_0^\infty \exp\left(-\rho t\right) \left\{ \sum_j \kappa_{ij} \left(\pi_{j,t} - x_{j,t}^2 \right) \right\} dt \right]$$

while the state evolves according to Equation (11). Here, we utilize the fact that the interest rate is derived from the household's Euler equation $r_t = \rho$. The dynamic problem gives the following HJB equation:

$$\rho V^i(z) = \max_{x_i} \left\{ z^\top Q^i z - \sum_j \kappa_{ij} x_{j,t}^2 + V_z^i(z) \left[(\mathbf{\Omega} - \delta I) z + \mu x \right] + \frac{\gamma^2}{2} z^\top V_{zz}^i(z) z \right\}. \tag{24}$$

The first term on the right-hand side of the HJB equation represents gross profit. The second term represents R&D costs. Therefore, the first two terms represent the firm's net instantaneous profit. The third term reflects the impact of changes in knowledge capital due to technology spillovers and R&D on firm i's value. The fourth term represents the expected effect of all firms' idiosyncratic shocks on firm i's value.

This dynamic game is a linear quadratic differential game, which significantly simplifies the analysis. This simplification is achieved because the following guess-and-verify approach is valid. Guess that the value can be expressed as a quadratic form of knowledge capital:

$$V^i(z) = z^\top X^i z. \tag{25}$$

The first and second derivatives of the value function are then given by:

$$V_z^i(z) = 2 z^\top X^i \tag{26}$$

$$V_{zz}^i(z) = 2 X^i \tag{27}$$

In a linear quadratic game, firms' strategies are expressed as linear functions of the state variables. Let X_i^i denote the i-th column of X^i. The first order condition with respect to x_i gives

$$x_i = \left(\mu X_i^i \right)^\top z. \tag{28}$$

Given firms' linear strategies, the law of motion for knowledge capital z_t can be rewritten as a geometric Brownian motion with constant coefficients. Define $\widetilde{X}$ as the matrix obtained by stacking X_i^i:

$$\widetilde{X} \equiv \begin{bmatrix} X_1^1 & \cdots & X_n^n \end{bmatrix}^\top \tag{29}$$

Then, the law of motion is rewritten as

$$dz_t = \mathbf{\Phi} z_t dt + \gamma \text{diag}(z_t) dW_t \tag{30}$$

where the matrix $\mathbf{\Phi}$ that captures the drift of the stochastic process is an $n \times n$ matrix such that

$$\boldsymbol{\Phi} \equiv \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{n1} & \phi_{n2} & \cdots & \phi_{nn} \end{bmatrix} \equiv \boldsymbol{\Omega} + \mu^2 \widetilde{\boldsymbol{X}} - \delta \boldsymbol{I}.$$

We refer to $\boldsymbol{\Phi}$ as the *technology transition matrix*, which consists of the exogenous technology-spillover matrix $\boldsymbol{\Omega}$, obsolescence at rate δ, and the endogenous R&D strategy-profile matrix $\mu^2 \widetilde{\boldsymbol{X}}$.

The HJB equation can be transformed into an algebraic Riccati equation, enabling the dynamic oligopoly problem with many firms to be solved in a computationally feasible timeframe. Inserting Equations (25) to (28) into the HJB equation Equation (24) and rearranging (see Section A.4), we obtain stacked algebraic Riccati equations

$$0 = \boldsymbol{Q}^i - \mu^2 \sum_j \kappa_{ij} \boldsymbol{X}_j^j \left(\boldsymbol{X}_j^j\right)^\top + \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}\right)^\top \boldsymbol{X}^i + \boldsymbol{X}^i \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}\right) \tag{31}$$

for $i = 1, 2, ..., n$. Note that the terms $\mu^2 \sum_j \kappa_{ij} \boldsymbol{X}_j^j \left(\boldsymbol{X}_j^j\right)^\top$ and $\boldsymbol{\Phi}$ depend on the $\boldsymbol{X}^j$ of other firms $j \neq i$. Therefore, to solve for $\boldsymbol{X}^i$, we need to simultaneously solve the algebraic Riccati equations for all firms.

Here, we define the stability of the solution.

Definition 1. The solution of the stacked algebraic Riccati equations Equation (31) is *stable* if all eigenvalues of $\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}$ have strictly negative real parts (i.e., the matrix is Hurwitz).

The requirement for stability of the solution corresponds to the condition commonly imposed in growth theory, which requires the discount rate to be sufficiently large to ensure that the utility over an infinite horizon is finite. In the following, we assume the existence of such a solution.

Assumption 1. *There exists a stabilizing solution* $\left(X^1, X^2, \ldots, X^n\right)$ *of the stacked algebraic Riccati equations (31).*

2.2.4 Balanced Growth Path in Deterministic Economy

In this subsection, we characterize the existence of a balanced growth path and its equilibrium in a deterministic economy using the technology transition matrix, $\boldsymbol{\Phi}$. We

Table 1: Growth Rate of Variables in Deterministic Economy

Growth Rate	Variable
0	$l_{i,t}$
g	$a_{i,t}$, $b_{i,t}$, $z_{i,t}$, $q_{i,t}$, $x_{i,t}$, $p_{i,t}/P_t$
$2g$	$\pi_{i,t}$, C_t, Y_t, w_t/P_t

Note: Growth rates are evaluated along the balanced-growth-path equilibrium in the deterministic economy (no shocks).

impose the following assumption on $\mathbf{\Phi}$, which is sufficient for the existence of a balanced-growth-path equilibrium:

Assumption 2. $\mathbf{\Phi}$ *is irreducible and Metzler; that is, its off-diagonal elements are nonnegative.*

One definition of irreducibility is as follows. Consider a directed graph with n vertices labeled $1, ..., n$, and an edge from vertex j to vertex i when $\phi_{ij} > 0$. Then $\mathbf{\Phi}$ is irreducible if and only if its associated graph is strongly connected (i.e., every vertex is reachable from every other vertex). The Metzler condition ensures that the deterministic law of motion preserves the nonnegative orthant. Intuitively, the assumption requires that all firms are connected, directly or indirectly, through the technology transition matrix.

Here, we define the balanced growth path equilibrium in our economy.

Definition 2. Balanced Growth Path Equilibrium: A *balanced-growth-path equilibrium* is a linear Markov perfect equilibrium such that the productivity of all firms grows at the same constant rate.

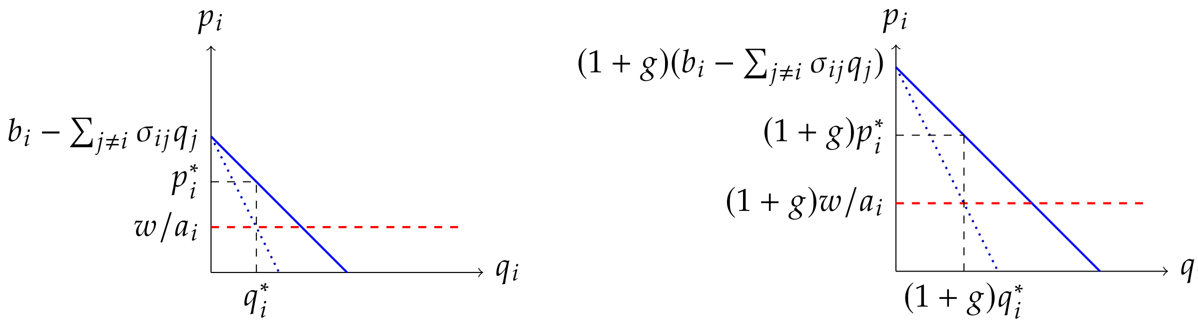
In the following proposition, we present sufficient conditions for the existence of a balanced growth path and characterize the equilibrium.

Proposition 1. *Let Assumptions 1 and 2 hold. Then* $\mathbf{\Phi}$ *has a real dominant eigenvalue, denoted* g*, and a strictly positive eigenvector* $\mathbf{z}$ *associated with* g*. Moreover, in the absence of shocks (*$\gamma = 0$*), for any strictly positive initial productivity vector, the normalized productivity distribution converges to a scalar multiple of* $\mathbf{z}$ *and all firms' productivities grow asymptotically at rate* g*.*

Proof. The proposition follows from the Perron-Frobenius theorem. □

Proposition 1 implies that the balanced-growth productivity distribution is uniquely characterized up to scale when the technology transition matrix satisfies Assumption 2. It also implies that the equilibrium productivity growth rate is given by the dominant eigenvalue of $\mathbf{\Phi}$, and the long-run productivity distribution is characterized by the associated eigenvector. Recall that the technology transition matrix consists of an exogenous

Figure 1: Partial Equilibrium Diagram and Its Proportional Growth



Note: The left panel depicts the static equilibrium for a firm's product, showing the residual demand (blue), marginal revenue (dotted line), and marginal cost (red) curves. The right panel illustrates how the equilibrium price, quantity, marginal cost, and the intercept of the residual demand scale proportionally with the growth factor $(1 + g)$.

technology-spillover matrix and an endogenous R&D strategy-profile matrix. Therefore, this result indicates that our model is an endogenous growth model that achieves a balanced growth path. The growth rates of variables on the balanced growth path are summarized in Table 1.

Figure 1 illustrates a partial equilibrium in a deterministic economy to provide an intuitive understanding of how a balanced-growth-path equilibrium is achieved. This diagram depicts the residual demand (blue), marginal revenue (dotted line), and marginal cost (red). In equilibrium, as quality (b_i), quantity (q_i), price (p_i), and marginal cost (w/a_i) all grow at the same rate g, the partial-equilibrium diagram expands homothetically at this common rate along both axes. Consequently, ratios such as revenue to cost and consumer surplus to producer surplus remain constant along the path. This constancy allows the economy to achieve balanced growth. The figure also suggests that growth in both labor productivity a_i and quality b_i is necessary. For example, if only quality b_i grows and labor productivity a_i does not, it is impossible for $q_i = a_i l_i$ to exhibit sustained growth because labor supply is inelastic.

Moreover, Figure 1 indicates why demand elasticity and the elasticity of substitution across goods remain constant along the balanced growth path. Because we assume a linear inverse demand system rather than CES demand, these elasticities would generally vary as the equilibrium point moves along the demand curve. Along the balanced growth path, however, the partial-equilibrium diagram scales proportionally, so the relative position of the equilibrium point on the demand curve does not change. As a result, both demand elasticity and the elasticity of substitution remain fixed throughout the path.

2.2.5 Social Welfare

Once the equilibrium is solved, household utility can be computed for any initial distribution at minimal additional computational cost. Let $V(z_0)$ denote the expected value of the representative household in competitive equilibrium, given initial distribution z_0:

$$V(z_0) = \boldsymbol{E}_0 \left[\int_0^\infty \exp(-\rho t) \left\{ Y_t - \sum_i x_{i,t}^2 \right\} dt \middle| z_0 \right].$$

Total output is

$$Y_t = \boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t \tag{32}$$

where

$$\boldsymbol{Q} = \frac{1}{2} \boldsymbol{N}^\top \left(2 \frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{\Sigma} \circ \left(\boldsymbol{K} + \boldsymbol{K}^\top \right) \right) \boldsymbol{N}$$

(see Section A.3 for the derivation). Ownership affects this quadratic form through the competitive-equilibrium allocation of quantities and qualities. Since equilibrium R&D is $\boldsymbol{x} = \mu \widetilde{\boldsymbol{X}} \boldsymbol{z}$, total R&D cost is $\boldsymbol{z}^\top \mu^2 \widetilde{\boldsymbol{X}}^\top \widetilde{\boldsymbol{X}} \boldsymbol{z}$. The representative household's HJB equation is:

$$\rho V(\boldsymbol{z}) = \boldsymbol{z}^\top \left(\boldsymbol{Q} - \mu^2 \widetilde{\boldsymbol{X}}^\top \widetilde{\boldsymbol{X}} \right) \boldsymbol{z} + V_z(\boldsymbol{z}) \boldsymbol{\Phi} \boldsymbol{z} + \frac{\gamma^2}{2} \boldsymbol{z}^\top V_{zz}(\boldsymbol{z}) \boldsymbol{z}$$

The first term on the right-hand side of the HJB equation represents the household's instantaneous consumption, equal to output net of total R&D cost. The second and third terms represent the expected change in the household's value due to the stochastic process of knowledge capital.

Since the household's problem is linear-quadratic, the value function takes a quadratic form in knowledge capital $\boldsymbol{z}$:

$$V(\boldsymbol{z}) = \boldsymbol{z}^\top \boldsymbol{X} \boldsymbol{z}.$$

We obtain the following (algebraic) Lyapunov equation by substituting $V(\boldsymbol{z})$ and its derivatives (see Section A.4):

$$0 = \boldsymbol{Q} - \mu^2 \widetilde{\boldsymbol{X}}^\top \widetilde{\boldsymbol{X}} + \boldsymbol{X} \left(\boldsymbol{\Phi} - \frac{1}{2} \left(\rho - \gamma^2 \right) \boldsymbol{I} \right) + \left(\boldsymbol{\Phi} - \frac{1}{2} \left(\rho - \gamma^2 \right) \boldsymbol{I} \right)^\top \boldsymbol{X} \tag{33}$$

Therefore, $\boldsymbol{X}$ is obtained as the solution to the Lyapunov equation Equation (33), and competitive-equilibrium welfare given the initial productivity vector $\boldsymbol{z}_0$ is $V(\boldsymbol{z}_0) = \boldsymbol{z}_0^\top \boldsymbol{X} \boldsymbol{z}_0$. This result indicates that, once the equilibrium is solved, we can compute the representative household's expected utility for any given initial distribution $\boldsymbol{z}_0$.

2.2.6 Total Value of Firms

Changes in ownership structure affect not only social welfare but also the division of income between capital and labor. To quantify how ownership concentration favors capital income over labor income, we characterize the total value of firms. Let V^P denote the total value of firms in competitive equilibrium, defined as:

$$V^P(z_0) = \boldsymbol{E}_0\left[\int_0^\infty \exp(-\rho t)\sum_i \left(\pi_{i,t} - x_{i,t}^2\right)dt \middle| z_0\right].$$

Aggregate real profits can be written in vector form as

$$\begin{aligned}\sum_i \pi_{i,t} &= \sum_i\sum_j \kappa_{ij}\sigma_{ij}q_{i,t}q_{j,t}\\ &= \boldsymbol{q}_t^\top(\boldsymbol{K}\circ\boldsymbol{\Sigma})\boldsymbol{q}_t\\ &= \boldsymbol{z}_t^\top\boldsymbol{P}\boldsymbol{z}_t\end{aligned}$$

where

$$\boldsymbol{P} = \frac{1}{2}\boldsymbol{N}^\top\left(\boldsymbol{\Sigma}\circ\left(\boldsymbol{K}+\boldsymbol{K}^\top\right)\right)\boldsymbol{N}.$$

By analogy with the social-welfare calculation in Section 2.2.5, V^P is obtained by solving the Lyapunov equation Equation (33) with $\boldsymbol{Q}$ replaced by $\boldsymbol{P}$. Denote the solution by $\boldsymbol{X}^P$. Then the total value of firms is

$$V^P(z_0) = \boldsymbol{z}_0^\top\boldsymbol{X}^P\boldsymbol{z}_0.$$

2.3 Counterfactual Scenarios

We examine several counterfactual scenarios to compare against the allocation under baseline common ownership. First, we analyze the allocation chosen by a benevolent social planner maximizing social welfare. Second, we consider the allocation resulting from profit maximization by firms under fully dispersed ownership. Finally, we explore the monopoly case, in which a single investor owns all firms—or, equivalently in our setting, all investors hold identical portfolios of firms.

2.3.1 Social Planner

To analyze the socially optimal allocation of R&D and its impact on economic growth and social welfare, we formulate the social planner's problem. The social planner aims to maximize the representative household's lifetime utility, subject to technological constraints.

As in the competitive equilibrium, the planner's problem decomposes into a static and a dynamic component.

Static Problem First, we characterize the static allocation. Given knowledge capital $\boldsymbol{z}_t$, the planner maximizes final-good production in each period:

$$\max_{\boldsymbol{a}_t,\boldsymbol{b}_t,\boldsymbol{l}_t,\boldsymbol{q}_t} Y_t = \boldsymbol{q}_t^\top \boldsymbol{b}_t - \frac{1}{2}\boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t,$$

subject to

$$\begin{aligned} q_{i,t} &= a_{i,t} l_{i,t} \quad i \in \{1,2,...,n\}, \\ z_{i,t} &= \zeta a_{i,t} + b_{i,t} \quad i \in \{1,2,...,n\}, \\ L &= \sum_i l_{i,t}. \end{aligned} \tag{34}$$

The first constraint is each firm's production technology; the second allocates each firm's knowledge capital between labor productivity and product quality; the third clears the labor market. Solving the planner's static problem yields quantities linear in knowledge capital:

$$\boldsymbol{q}_t^* = \boldsymbol{N}^* \boldsymbol{z}_t$$

and output as a quadratic form in knowledge capital:

$$Y_t^* = \frac{1}{2}\boldsymbol{z}_t^\top \boldsymbol{N}^* \boldsymbol{z}_t$$

where

$$\boldsymbol{N}^* = \left(2\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma}\right)^{-1}$$

(see Section A.5).

Dynamic Problem Next, we characterize the dynamic allocation in the social planner's problem. The social planner's dynamic problem is formulated as follows:

$$V^*(\boldsymbol{z}_0) \equiv \max_{\{\boldsymbol{x}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty \exp(-\rho t) \left\{\frac{1}{2}\boldsymbol{z}_t^\top \boldsymbol{N}^* \boldsymbol{z}_t - \boldsymbol{x}_t^\top \boldsymbol{x}_t\right\} dt\right],$$

subject to the law of motion of knowledge capital (11). The HJB equation of the social planner's value V^* derived from the dynamic problem is given by

$$\rho V^*(\boldsymbol{z}) = \max_{\boldsymbol{x}\geq 0} \left\{ \frac{1}{2} \boldsymbol{z}^\top \boldsymbol{N}^* \boldsymbol{z} - \boldsymbol{x}^\top \boldsymbol{x} + V_{\boldsymbol{z}}^*(\boldsymbol{z}) \left[(\boldsymbol{\Omega} - \delta \boldsymbol{I}) \boldsymbol{z} + \mu \boldsymbol{x} \right] + \frac{\gamma^2}{2} \boldsymbol{z}^\top V_{\boldsymbol{zz}}^*(\boldsymbol{z}) \boldsymbol{z} \right\} \tag{35}$$

The first term on the right-hand side of the HJB equation represents output. The second term represents the total R&D cost. Therefore, the first two terms represent the household's instantaneous consumption. The third and fourth terms represent the expected change in the household's value due to the stochastic process of knowledge capital.

Since the social planner's problem is also linear quadratic, household utility can be expressed in a quadratic form of knowledge capital $\boldsymbol{z}$. Guess that the value is expressed as a quadratic form of knowledge capital:

$$V^*(\boldsymbol{z}) = \boldsymbol{z}^\top \boldsymbol{X}^* \boldsymbol{z} \tag{36}$$

The socially optimal allocation of R&D is a linear function of knowledge capital $\boldsymbol{z}$, as in the competitive equilibrium:

$$\boldsymbol{x} = \mu \boldsymbol{X}^* \boldsymbol{z} \tag{37}$$

Therefore, again, the law of motion for knowledge capital $\boldsymbol{z}_t$ is rewritten as a geometric Brownian motion with constant coefficients:

$$d\boldsymbol{z}_t = \boldsymbol{\Phi}^* \boldsymbol{z}_t dt + \gamma \text{diag}(\boldsymbol{z}_t) d\boldsymbol{W}_t$$

where the optimal technology transition $\boldsymbol{\Phi}^*$ is given by

$$\boldsymbol{\Phi}^* \equiv (\boldsymbol{\Omega} - \delta \boldsymbol{I}) + \mu^2 \boldsymbol{X}^*$$

Inserting (36) and its first and second derivatives with respect to $\boldsymbol{z}$ and (37) into (35), we obtain a single algebraic Riccati equation:

$$0 = \frac{1}{2} \boldsymbol{N}^* - \mu^2 (\boldsymbol{X}^*)^2 + \boldsymbol{X}^* \left(\boldsymbol{\Phi}^* - \frac{1}{2} (\rho - \gamma^2) \boldsymbol{I} \right) + \left(\boldsymbol{\Phi}^* - \frac{1}{2} (\rho - \gamma^2) \boldsymbol{I} \right) \boldsymbol{X}^* \tag{38}$$

Therefore, $\boldsymbol{X}^*$ is obtained as the solution of the algebraic Riccati equation (38). Remember that, in the competitive equilibrium, a Riccati equation exists for each firm, requiring simultaneous solutions of these equations. In contrast, for the social planner problem, we only need to solve a single Riccati equation, making the computation much simpler.

The social planner's problem yields a quadratic form for social welfare under the optimal allocation: $V^*(\boldsymbol{z}_0) = \boldsymbol{z}_0^\top \boldsymbol{X}^* \boldsymbol{z}_0$. This result allows us to efficiently calculate the expected utility of the representative household for any initial distribution $\boldsymbol{z}_0$, requiring minimal additional computation once $\boldsymbol{X}^*$ is obtained.

2.3.2 Dispersed Ownership

As the second counterfactual scenario, we consider the case in which firm ownership does not overlap across investors. In this case, maximizing investor returns is equivalent to each firm maximizing its own profit. This corresponds to the common-ownership weight matrix being the identity, i.e., $\boldsymbol{K} = \boldsymbol{I}$.

2.3.3 Monopoly

As the third counterfactual scenario, we consider the monopoly case, in which a single investor owns all firms, to analyze the allocation of R&D, its impact on economic growth, and social welfare under an extremely concentrated ownership structure. In our setting—where firms maximize a weighted sum of investors' returns—the monopoly case is equivalent to multiple investors holding identical portfolios of firm ownership. This corresponds to a common-ownership matrix of ones, $\boldsymbol{K} = \boldsymbol{J}$. Because all firms share the same objective function, the equilibrium is characterized by a single Riccati equation. For details on the characterization and solution of the equilibrium, see Section A.6.

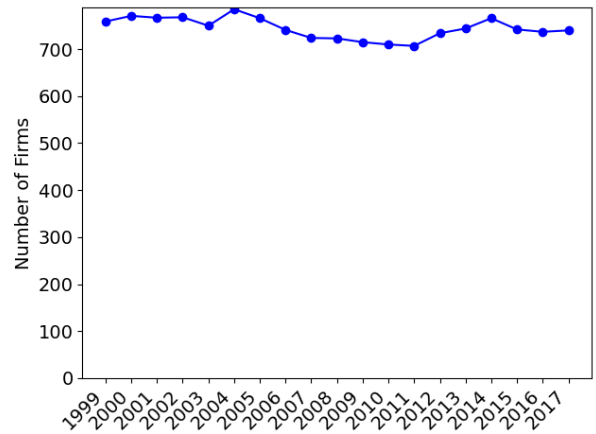
2.3.4 Different Ownership Structures for Production and R&D

We consider counterfactuals in which the ownership structure relevant for static production differs from that for dynamic R&D, to better understand how ownership affects firms' R&D decisions. For example, take the static competitive equilibrium under baseline common ownership as given and, conditional on it, let a benevolent social planner choose the R&D allocation. Starting from an initial productivity distribution $\boldsymbol{z}_0$, the planner solves

$$V^{**}(\boldsymbol{z}_0) \equiv \max_{\{\boldsymbol{x}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty \exp(-\rho t) \left\{ \boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t - \boldsymbol{x}_t^\top \boldsymbol{x}_t \right\} dt \right],$$

subject to the law of motion Equation (11). Unlike in Section 2.3.1, $\boldsymbol{Q}$ replaces $\boldsymbol{N}^*/2$ in the planner's objective. Accordingly, the constrained planner's allocation is obtained by solving the modified algebraic Riccati equation Equation (38) with $\boldsymbol{N}^*/2$ replaced by $\boldsymbol{Q}$.

Figure 2: Number of Firms



Note: This figure plots the number of sample firms by year. The potential set of firms consists of U.S. publicly traded firms that appear in both DISCERN 2 (which links patents to public firms) and the Common Ownership Data from Backus et al. (2021). From this set, we retain firms with positive profit (revenue minus cost of goods sold) and at least ten patents in the year or within two years on either side (a five-year window).

Table 2: Model Parameters

Notation	Description	Value	Source
$\mathbf{\Psi}^\top\mathbf{\Psi}$	Product proximity		Form 10-K, Hoberg and Phillips (2016)
$\widetilde{\mathbf{\Omega}}$	Technological proximity		USPTO patent classifications
$\boldsymbol{K}$	Common ownership weights		Form 13F, Backus et al. (2021)
α	Product proximity → Substitutability	0.12	Pellegrino (2024)
β	Technological proximity → Spillover	0.024	Estimated from the law of motion
γ	Std. dev. of idiosyncratic shocks	0	Deterministic economy in the baseline
ζ/L	Labor-augmentation efficiency	0.0063	Compustat, cost of goods sold
ρ	Discount rate	0.10	> risk-free rates, < private R&D returns
μ	R&D efficiency	0.066	2.6% R&D share (moment match)
δ	Depreciation rate	0.017	1.2% growth rate (moment match)

3 Data and Estimation

We map the model parameters—including three networks (product proximity, technology proximity, and ownership structure)—to observed data. This section describes the data and the estimation procedure. Figure 2 plots the number of sample firms by year, and Table 2 summarizes the estimated parameters and their sources.

3.1 Sample Construction

The sample is an unbalanced panel of U.S. publicly traded firms from 1999 to 2017. We begin with firms that can be linked across Compustat, DISCERN 2, the Hoberg–Phillips product-similarity data, and the common-ownership data of Backus et al. (2021). This restriction ensures that, for each firm-year used in the quantitative model, we observe accounting variables, patent-based technology classes, product-market proximity, and institutional ownership.

We then impose two additional restrictions. First, we retain firm-years with positive gross profits, measured as revenue minus cost of goods sold, because the model's static profit identity is used to recover positive quantities. Second, we require at least ten patents in the year or within two years on either side. This five-year window reduces noise in patent-class shares while preserving the annual timing of the panel. The resulting panel contains more than 700 firms in each year, as shown in Figure 2. Since firm entry and exit from the sample are allowed, all network matrices and firm-level objects are constructed year by year on the set of firms observed in that year.

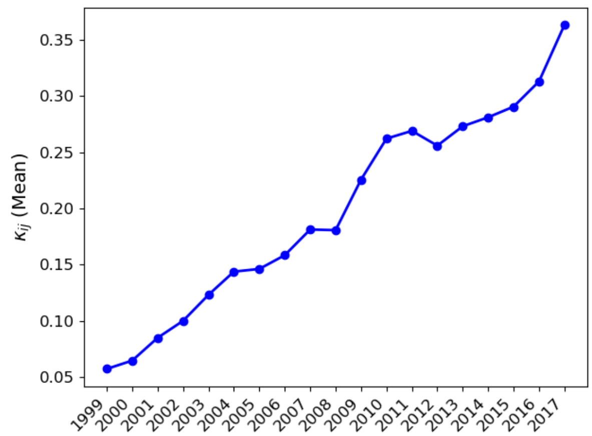
3.2 Common Ownership Weight

To calculate the matrix of common ownership weights $\boldsymbol{K}$, we first construct each firm's ownership-share vector $\boldsymbol{s}_i$. We build $\boldsymbol{s}_i$ from a dataset compiled by Backus et al. (2021), who constructed the holdings from 13F filings. Form 13F is a mandatory filing with the Securities and Exchange Commission (SEC) that requires institutional investors with over \$100 million in assets under management to disclose their holdings of U.S. securities, including those issued by all U.S. public corporations.

Using CUSIP codes, we merge this dataset with Compustat's total shares outstanding for each firm. We then divide each investor's holdings in a firm by that firm's total shares outstanding to obtain the normalized ownership share for each investor–firm pair. Collecting these values across all investors for a given firm produces the normalized ownership-share vector $\boldsymbol{s}_i$. Finally, we apply Equation (13) to compute the common ownership weights matrix $\boldsymbol{K}$.

Following Ederer and Pellegrino (2024), we incorporate a statistical correction to account for unobserved investors. When corporate ownership is not fully observed, the denominator in Equation (13) is biased downward, leading to unrealistically large κ_{ij} values for certain firm pairs. To address this, we assume that unobserved ownership is similar to observed ownership. Specifically, let S_i^O denote the total fraction of firm i's shares held by observed investors, and let $\left(\boldsymbol{s}_i^\top \boldsymbol{s}_i\right)^O$ denote the scalar computed using only observed ownership. We

Figure 3: Mean κ_{ij}



Note: This figure plots the average value of the common ownership weight κ_{ij} for all pairs of sample firms from 1999 to 2017. Following Ederer and Pellegrino (2024), a statistical adjustment is applied in the calculation of the denominator of κ_{ij} to address the bias caused by unobserved ownership.

then adjust the denominator of κ_{ij} as follows:

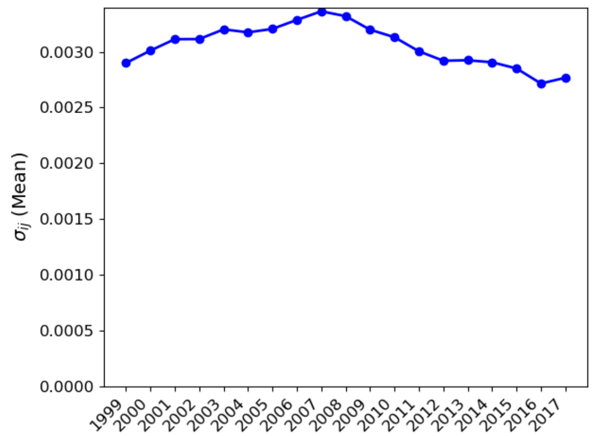
$$\boldsymbol{s}_i^\top \boldsymbol{s}_i = \left[1 + \left(\frac{1 - S_i^O}{S_i^O}\right)^2\right] \left(\boldsymbol{s}_i^\top \boldsymbol{s}_i\right)^O$$

As shown in Figure 3, the average common ownership weight κ_{ij} increased roughly sevenfold from 1999 to 2017, reflecting the rise of index funds and increased investor concentration. Within our framework, this implies that firms increasingly took into account the values of other firms when determining production and R&D activity during this period.

3.3 Product Market Proximity and Substitutability

Following Pellegrino (2024), we measure product-market proximity, denoted by $\boldsymbol{\Psi}^\top \boldsymbol{\Psi}$, using the dataset developed by Hoberg and Phillips (2016). The measure is based on the cosine similarity of words appearing in the Business Description sections of 10-K filings. Hoberg and Phillips (2016) constructed a vocabulary of 61,146 words used by firms to describe the features and characteristics of their products. For each firm i, they generated a vector of word frequencies; each element records the number of times a given word appears in the firm's product description. Their analysis shows that cosine similarity effectively identifies industry groupings and predicts competitive relationships between

Figure 4: Mean σ_{ij}



Note: This figure plots the cross-firm mean of the product proximity σ_{ij} by year, 1999–2017. Following Pellegrino (2024), we construct σ_{ij} using the Hoberg and Phillips (2016) text-based product similarity—cosine similarity of the 10-K Business Description. The mean is the simple average over all $i \neq j$ pairs in the sample each year.

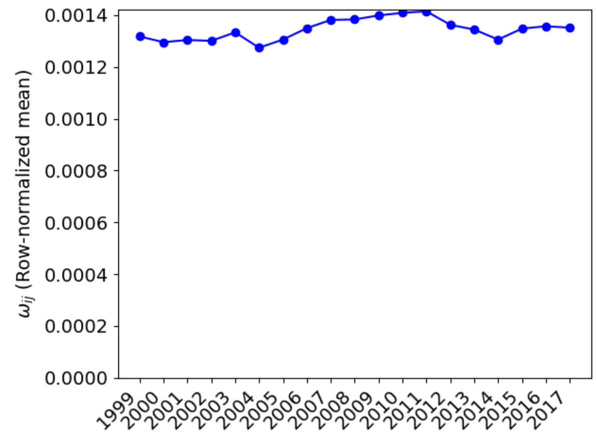
firms, outperforming alternative industry classifications. Figure 4 plots the average product proximity across all firm pairs from 1999 to 2017 and shows no clear trend over this period.

We take α, which governs horizontal differentiation, from Pellegrino (2024). He estimates α to match the inverse of the cross-price elasticity of demand documented by Nevo (2001), since this parameter maps cosine similarity between products into the magnitude of cross-price elasticities. Pellegrino (2024) show that price elasticities implied by the Generalized Hedonic Linear demand system align well with estimates in the IO literature. Specifically, he compares elasticities in automobile (Berry et al., 1995), ready-to-eat cereal (Nevo, 2001), and computer markets (Goeree, 2008). Table 2 in Pellegrino (2024) indicates that the magnitudes of own-price elasticities by firm and cross-price elasticities between firms, as implied by the Generalized Hedonic Linear demand system, are similar to those reported in the literature.

3.4 Technological Proximity

For technological proximity, we use patent data and follow the methodologies of Bloom et al. (2013) and Lucking et al. (2019) to construct measures of technological proximity between firms. Specifically, we utilize the DISCERN 2 dataset (Arora et al., 2024), which links data on U.S. publicly listed firms from the Compustat database to their patents. Following Bloom et al. (2013), we calculate technological proximity between firms using the Jaffe measures based on the Cooperative Patent Classification.

Figure 5: Mean $\widetilde{\omega}_{ij}$



Note: This figure plots the average value of the technological proximity $\widetilde{\omega}_{ij}$ for all pairs of sample firms from 1999 to 2017. The proximity is calculated using the Jaffe measure based on group-level patent classifications.

Denote by T_i the vector of firm i's patent shares across technology classes. Under the Jaffe measure (Jaffe, 1986), technological proximity is given by

$$\hat{\omega}_{ij} = \frac{T_i^\top T_j}{\left(T_i^\top T_i\right)^{1/2}\left(T_j^\top T_j\right)^{1/2}}.$$

In our baseline, we construct T_i using group-level Cooperative Patent Classifications. Similar to Liu and Ma (2024), we normalize the spillover weights so that they sum to one for each firm:

$$\tilde{\omega}_{ij} = \frac{\hat{\omega}_{ij}}{\sum_{k\neq i}\hat{\omega}_{ik}}, \qquad j \neq i.$$

Figure 5 plots the average value of the technological proximity $\widetilde{\omega}_{ij}$ for all pairs of sample firms, calculated using the Jaffe measure with group-level patent classifications.

3.5 Knowledge Capital Distribution and Labor-Augmentation Efficiency

Next, we identify firms' profits $\{\pi_{i,t}\}$, quantities $\boldsymbol{q}_t$, labor-augmentation efficiency ζ/L, and knowledge capital $\boldsymbol{z}_t$ step-by-step. The resulting $\boldsymbol{z}_t$ is a model-implied measure of knowledge capital. It is not a perpetual-inventory R&D stock or a direct patent count; instead, it is the level of firm knowledge capital that rationalizes observed profits, production costs, and the measured product-market and ownership networks through the model's static equilibrium conditions. This model-implied $\boldsymbol{z}_t$ is then used for estimating the law of motion and as an initial distribution for numerical analysis.

The firm-level profit $\pi_{i,t}$ is measured as revenue (revt) minus cost of goods sold (cogs) in the Compustat database. Nominal variables such as revenue and cost of goods sold are deflated using the GDP deflator. Given product-market rivalry σ_{ij} and ownership structure κ_{ij}, we identify the quantity vector $\boldsymbol{q}_t$ so that it satisfies the following profit identity for each firm implied by the model:

$$\pi_{i,t} = \sum_j \kappa_{ij}\sigma_{ij}q_{i,t}q_{j,t}$$

This is a system of nonlinear equations in $\boldsymbol{q}_t$. Since $\kappa_{ii} = \sigma_{ii} = 1$, each equation can be written as

$$\pi_{i,t} = q_{i,t}^2 + q_{i,t}\sum_{j\neq i}\kappa_{ij,t}\sigma_{ij,t}q_{j,t}.$$

We solve this system by fixed-point iteration on the positive root,

$$q_{i,t} = \frac{-\sum_{j\neq i}\kappa_{ij,t}\sigma_{ij,t}q_{j,t} + \sqrt{\left(\sum_{j\neq i}\kappa_{ij,t}\sigma_{ij,t}q_{j,t}\right)^2 + 4\pi_{i,t}}}{2},$$

starting from a positive vector. This procedure recovers quantities in the model's units for each year. We then identify ζ/L using the following relationship in our model:

$$\frac{\zeta}{L}\boldsymbol{q}_t^\top \boldsymbol{J}\boldsymbol{q}_t = \text{total production cost at time } t.$$

We calculate the total production cost as the sum of the costs of goods sold (cogs) for all firms in our sample. Note that in our model, ζ and L cannot be separately identified; only their ratio matters for equilibrium allocations. Finally, we identify the knowledge capital of firms $\boldsymbol{z}_t$ by inverting the static equilibrium condition Equation (23):

$$\boldsymbol{z}_t = \left(2\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma}_t + \boldsymbol{K}_t \circ \boldsymbol{\Sigma}_t\right)\boldsymbol{q}_t.$$

3.6 Law of Motion of Knowledge Capital

Using each firm's identified knowledge capital, we estimate the law of motion in our model to recover the technological spillover parameter β. Dividing the drift in Equation (11) by

Table 3: Estimates of the law of motion for knowledge capital

	(1)	(2)	(3)
Dependent variable:	$\Delta \log z_{i,t}$	$\Delta \log z_{i,t}$	$\Delta \log z_{i,t}$
$\sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}}$	0.026^{**}	0.024^{**}	0.073^{*}
	(0.010)	(0.010)	(0.041)
$\log z_{i,t}$	-0.132^{***}	-0.162^{***}	-0.088^{**}
	(0.013)	(0.013)	(0.038)
$\frac{x_{i,t}}{z_{i,t}}$		0.514^{***}	
		(0.064)	
Year fixed effects	✓	✓	✓
Firm fixed effects	✓	✓	✓
IV			✓
IV first-stage F-statistic			49.198
No. of observations	14,442	14,442	14,442
R^2	0.713	0.716	–
Within R^2	0.072	0.080	–

Note: The dependent variable is $\Delta \log z_{i,t} \equiv \log z_{i,t+1} - \log z_{i,t}$. The spillover regressor is the technological-proximity-weighted knowledge capital, $\sum_{j \neq i} \tilde{\omega}_{ij,t}\, z_{j,t}/z_{i,t}$, where $\tilde{\omega}_{ij,t}$ denotes technological proximity. Controls include firm and year fixed effects, $\log z_{i,t}$, and R&D intensity $x_{i,t}/z_{i,t}$ when indicated. Column (3) reports IV estimates using the instrument $\sum_{j \neq i} \tilde{\omega}_{ij,t}\, z_{j,t}^{\mathrm{TAX}}$, where $z_{j,t}^{\mathrm{TAX}}$ is constructed by cumulating predicted R&D over the prior five years based on tax-induced variation in the R&D user cost. Standard errors are two-way clustered by year and 4-digit NAICS industry. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

$z_{i,t}$ gives the empirical analogue

$$\Delta \log z_{i,t} \approx \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}} - \delta + \mu \frac{x_{i,t}}{z_{i,t}},$$

up to approximation error and shocks. The regression below is used to discipline β, while μ and δ are calibrated separately using aggregate moments. This separation is useful because Compustat R&D expenditure may omit some innovation inputs and because the mapping from reported R&D expenditure to the model's effort variable is governed by the model's quadratic R&D technology. Specifically, we estimate the following regression equation:

$$\log z_{i,t+1} - \log z_{i,t} = \beta \sum_{j \neq i} \tilde{\omega}_{ij,t} \frac{z_{j,t}}{z_{i,t}} + \text{Controls}_{i,t} + \epsilon_{i,t} \tag{39}$$

where $z_{i,t}$ denotes firm i's knowledge capital at time t, and $\tilde{\omega}_{ij,t}$ measures the technological proximity between firms i and j at time t. $\text{Controls}_{i,t}$ includes year and firm fixed effects, log knowledge capital $\left(\log z_{i,t}\right)$, and R&D intensity $(x_{i,t}/z_{i,t})$.

Table 3 reports estimates of Equation (39). In column (1), the coefficient on technological-proximity-weighted knowledge capital, $\sum_{j \neq i} \tilde{\omega}_{ij,t} z_{j,t}/z_{i,t}$, is positive and statistically significant, indicating that knowledge capital from technologically proximate firms contributes

Table 4: First stage: predicting R&D using tax credits

	R&D expenditure (1)
State tax credit component of R&D user cost	-1.163^{***} (0.294)
Federal tax credit component of R&D user cost	-34.298^{***} (3.649)
Firm fixed effects	✓
Year fixed effects	✓
No. of observations	16,197

Note: Outcome is firm-level R&D expenditure. Regressors are components of the R&D user cost based on federal and state tax credits, following Lucking et al. (2019), with firm and year fixed effects. This specification corresponds to the first stage used to construct predicted knowledge capital, $z_{j,t}^{\text{TAX}}$. Standard errors are two-way clustered by year and 4-digit NAICS industry. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

to a firm's own knowledge growth. Column (2) adds firms' R&D investment, $x_{i,t}$, as a control; the spillover coefficient remains positive and significant.

Column (3) addresses endogeneity using an instrumental-variables strategy that exploits tax-induced variation in the effective user cost of R&D, following Wilson (2009), Bloom et al. (2013), and Lucking et al. (2019). The concern is that common technology shocks may raise both a firm's own knowledge growth and the knowledge capital of technologically proximate firms, biasing the spillover coefficient upward. We first regress a firm's R&D expenditure on the user cost of R&D, constructed using tax credits in Lucking et al. (2019), to obtain predicted R&D. We then cumulate the prior five years of predicted R&D to construct predicted knowledge capital $z_{j,t}^{\text{TAX}}$ and form the instrument $\sum_{j\neq i} \tilde{\omega}_{ij,t} z_{j,t}^{\text{TAX}}$. The exclusion restriction is that tax-induced variation in other firms' predicted knowledge capital affects firm i's knowledge growth through technological spillovers, after controlling for firm and year fixed effects, rather than through firm i's own demand or productivity shocks. The IV estimate of the spillover coefficient remains positive and statistically significant.

3.7 Discount Rate and R&D Coefficient

The remaining parameters are the discount rate ρ, the R&D efficiency μ, and the depreciation rate δ. We set ρ to 10%. This value is above safe real interest rates and below typical private returns to R&D, and it also ensures that the infinite-horizon value functions are finite for the estimated transition matrix. Holding ρ fixed, we calibrate μ and δ to match two aggregate moments. Specifically, we choose these parameters so that (i) the model's R&D expenditure share, $\sum_i x_{i,t}^2/Y_t$, equals the business enterprise R&D share of GDP (2.6%);

and (ii) the model's implied annual growth rate matches the observed average growth rate of U.S. GDP per capita over 1999–2017 (1.2%).

In principle, μ could be estimated from the coefficient on $x_{i,t}/z_{i,t}$ in the law-of-motion regression. We do not use that coefficient as the structural value of μ because reported Compustat R&D expenditures may cover only part of the innovation inputs in the model, and because the model's effort variable is the square root of final-good R&D expenditure. Instead, the regression is used to identify the spillover strength β, while μ governs the aggregate scale of equilibrium R&D and is disciplined by the aggregate R&D share.

3.8 Goodness of Fit

Next, we evaluate the goodness of fit of the estimated model for both target and non-target moments. Figure 6 illustrates firm-level profits, sales, and R&D expenditures, comparing the model's output with actual data. Firm-level profits are matched mechanically through the recovery of quantities from the static profit identity, so panel (a) is best interpreted as a check on the numerical inversion. The more informative panels are sales and R&D expenditures, which are not directly targeted at the firm level. The correlations between the model and the data are 0.96 for the log of sales and 0.64 for the log of R&D expenditures. Even though these variables are not targets in the calibration, the model-implied and observed values remain highly correlated.

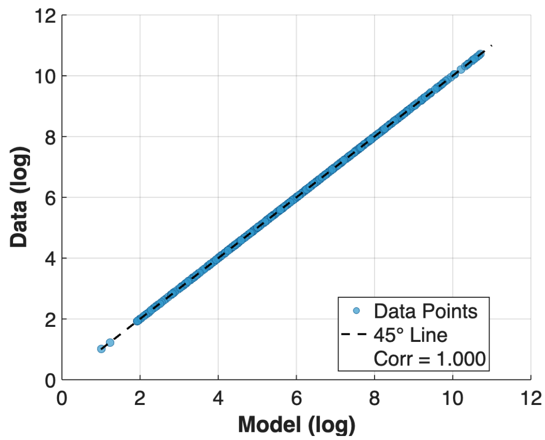
Figure 7 compares the growth rates of knowledge capital in the model and in the data. The data-based growth rate is computed from changes between 2010 and 2017. To construct the growth rate in the model, we use the 2010 network and the initial level of knowledge capital. The correlation between the model-implied and data-based growth rates of knowledge capital is 0.51, showing that the model-generated growth rates line up closely with those observed in the data.

3.9 Comparison with Anton et al. (2024)

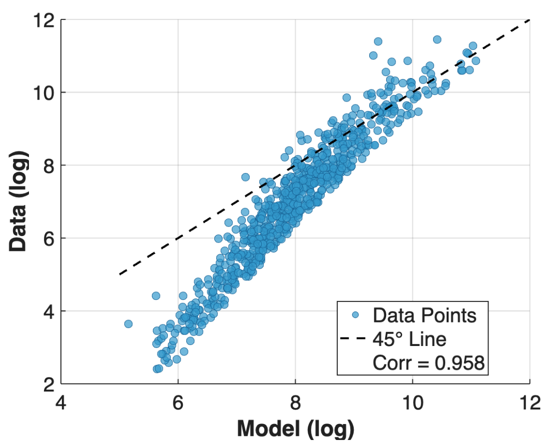
To further validate the model, we compare the regression coefficients implied by our simulated data with the reduced-form estimates of Anton et al. (2024). Their analysis uses U.S. firm-level data to estimate the impact of common ownership on innovation, focusing on whether firms internalize the business-stealing and technology-spillover forces. They

Figure 6: Firm-Level Profits, Sales, and R&D Expenditures: Model vs. Data

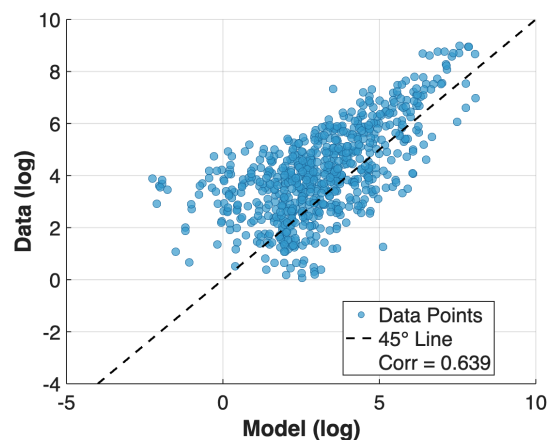
(a) log (Profits)



(b) log (Sales)

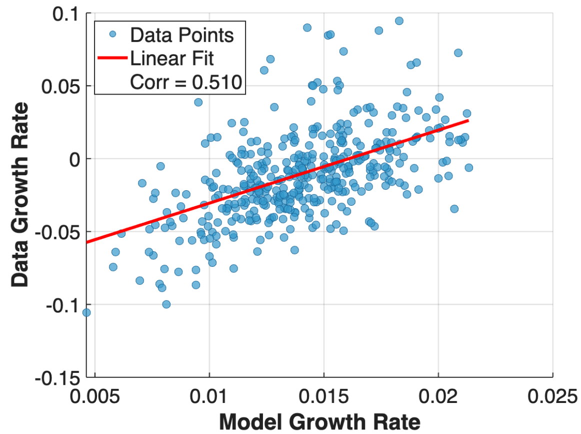


(c) log (R&D Expenditures)



Note: These panels plot firm-level profits, sales, and R&D expenditures from the model and from Compustat. Profits in the data are measured as "Revenue – Total" (revt) minus "Cost of Goods Sold" (cogs) in Compustat. The model is calibrated so that firm-level profits match those in the data.

Figure 7: Growth Rate Comparison: Model vs. Data



Note: This figure compares firm-level growth rates of knowledge capital implied by the model and computed from the data. The data-based growth rate is measured between 2010 and 2017 and is constructed using the 2010 network and the initial level of knowledge capital.

estimate the following regression:

$$\log\left(1+\frac{\text{R\&D}_{it}}{A_{it}}\right) = \gamma_1 \log\left(\sum_{j\neq i} \kappa_{ijt}\,\text{technology proximity}_{ijt}\, G_{jt}\right)$$

$$+\gamma_2 \log\left(\sum_{j\neq i} \kappa_{ijt}\,\text{product proximity}_{ijt}\, G_{jt}\right) \tag{40}$$

$$+\text{Controls}_{it} + \text{Firm FEs}_i + \text{Year FEs}_t + \varepsilon_{ijt}. \tag{41}$$

In this specification, A_{it} denotes firm i's total assets reported in Compustat, and G_{jt} is the R&D stock constructed by the perpetual inventory method. Technology and product proximities are constructed in the same way as we do in our model. Although their data sources differ from ours, the common ownership weights κ_{ijt} are computed under the same Rotemberg proportional influence assumption.[6] Additional controls include the average common ownership weight $\frac{1}{n-1}\sum_{j\neq i}\kappa_{ijt}$, the pool of product market spillovers $\sum_{j\neq i}\text{product proximity}_{ijt}G_{jt}$, the pool of technology market spillovers $\sum_{j\neq i}\text{technology proximity}_{ijt}G_{jt}$, the log of sales, the capital–labor ratio, and the firm's share of institutional ownership.

We reproduce this regression with simulated outcomes from our model, using the model's parameters and the implied endogenous variables. Whereas Anton et al. (2024)

[6]Anton et al. (2024) supplement the 13F filings that we use with block-holding information from 13D and 13G filings.

Table 5: Replication of Anton et al. (2024)

	Anton et al. (2024)	Our model
$\log\left(\sum_{j\neq i} \kappa_{ijt} \text{ tech proximity}_{ijt} G_{jt}\right)$	0.00513** (0.00226)	0.00194*** (0.000272)
$\log\left(\sum_{j\neq i} \kappa_{ijt} \text{ product proximity}_{ijt} G_{jt}\right)$	−0.00457** (0.00222)	−0.00547*** (0.000693)
Observations	31,169	14,102

Note: This table reports coefficients from panel regressions of firm-level log R&D intensity on common ownership-weighted proximity measures. Both specifications include firm and year fixed effects and control for the log of sales together with the average common ownership weight $\left(\frac{1}{n-1}\sum_{j\neq i}\kappa_{ijt}\right)$, the pool of product market spillovers $\left(\sum_{j\neq i} \text{product proximity}_{ijt} G_{jt}\right)$, and the pool of technology market spillovers $\left(\sum_{j\neq i} \text{technology proximity}_{ijt} G_{jt}\right)$. The Anton et al. (2024) column additionally controls for the capital–labor ratio and the firm's share of institutional ownership; our model column omits these controls because the model does not produce analogous statistics. Standard errors, shown in parentheses, are clustered at the firm level. The number of observations equals the firm-year pairs used in each analysis. Statistical significance is denoted by * $p < 0.10$, ** $p < 0.05$, and *** $p < 0.01$.

study the period 1985–2015, our data cover 1999–2017. Because the model does not generate firm assets, R&D stocks, or physical capital, we proxy for each of these objects with the firm-level knowledge capital z_{it}. Consistent with their specification, we include firm and year fixed effects and control for the log of sales along with the model-implied averages of common ownership weights, product proximity, and technology proximity. Unlike Anton et al. (2024), we exclude the capital–labor ratio and the firm's share of institutional ownership because the model does not produce comparable statistics.

Table 5 demonstrates that both the empirical estimates and our model generate a positive and statistically significant technology-spillover channel alongside a negative business-stealing channel. The coefficient on the interaction between the common ownership weight and technology proximity is slightly smaller in our model than in Anton et al. (2024), while the coefficient on the interaction between the common ownership weight and product proximity is larger in absolute value but remains in the same order of magnitude.

4 Numerical Analysis

In this section, we conduct several numerical exercises using the estimated model. The main exercise is to analyze the impact of ownership structure on economic growth and social welfare.

Table 6: Computational Complexity and Scale

Model	Computational complexity	Number of firms	Productivity state space
Cavenaile et al. (2023)	$O(2^n)$	4	Six grids
Our model	$O(n^4)$	707–785	Continuous

4.1 Algorithms and Computation Speed

To solve the competitive equilibrium, we solve the system of Riccati equations backward. Specifically, given $\begin{bmatrix} \boldsymbol{X}_\tau^1 & \cdots & \boldsymbol{X}_\tau^n \end{bmatrix}$, update $\begin{bmatrix} \boldsymbol{X}_{\tau-\Delta}^1 & \cdots & \boldsymbol{X}_{\tau-\Delta}^n \end{bmatrix}$ by

$$-\frac{\boldsymbol{X}_\tau^i - \boldsymbol{X}_{\tau-\Delta}^i}{\Delta} = \boldsymbol{Q}^i - \mu^2 \sum_j \kappa_{ij} \boldsymbol{X}_\tau^j \left(\boldsymbol{X}_\tau^j\right)^\top + \left(\boldsymbol{\Phi}_\tau - \frac{1}{2}\left(\rho - \gamma^2\right)\boldsymbol{I}\right)^\top \boldsymbol{X}_\tau^i + \boldsymbol{X}_\tau^i \left(\boldsymbol{\Phi}_\tau - \frac{1}{2}\left(\rho - \gamma^2\right)\boldsymbol{I}\right)$$

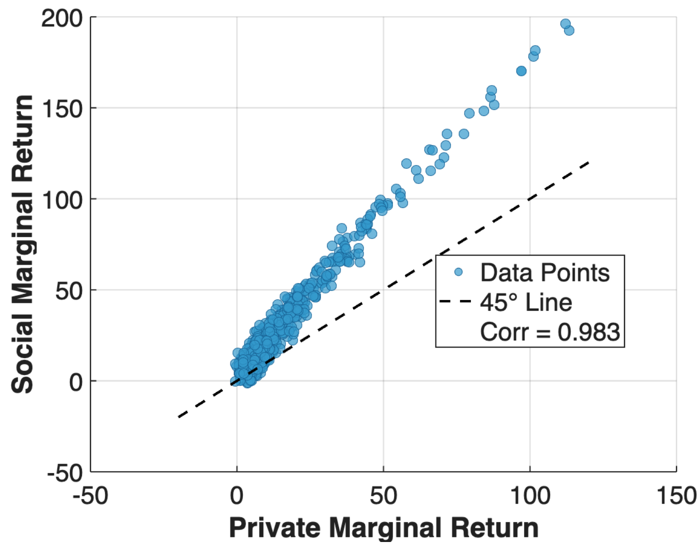
When the number of firms is n, each firm has an $n \times n$ matrix of undetermined coefficients $\boldsymbol{X}^i$. As shown in Figure 2, the number of sample firms ranges between 707 (in 2011) and 785 (in 2004). For $n = 700$, the total number of undetermined coefficients is $n^3 = 700^3 \simeq 343$ million. Despite this scale, an efficient vectorized algorithm allows us to solve the problem in a few minutes on a laptop.

Within the framework of existing endogenous economic growth models, solving the dynamic oligopoly problem involving such a large number of firms is challenging. When considering a dynamic oligopoly with n firms, the state variables typically include the productivity of each firm, leading to a computational complexity of $O\left(2^n\right)$. For instance, Cavenaile et al. (2023) introduce a dynamic oligopoly into a Schumpeterian growth model, similar to ours. However, due to the curse of dimensionality, they analyze oligopolies with a maximum of four firms. In contrast, our model has a computational complexity of order $O\left(n^4\right)$, allowing us to analyze markets with more than 700 firms. Furthermore, while Cavenaile et al. (2023)'s model uses only six discrete grids for productivity, our model allows for a continuous state space.

4.2 Social and Private Marginal Returns to R&D

We compare private and social marginal returns to R&D. We measure the private marginal return to R&D as the increase in the weighted value of a firm's owners induced by its R&D

Figure 8: Private and Social Marginal Returns on R&D



Note: Each point represents a firm. The horizontal axis reports the private marginal return on R&D, $2\mu\,\widetilde{\boldsymbol{X}}\boldsymbol{z}$; the vertical axis reports the social marginal return on R&D, $2\mu\,\boldsymbol{X}\boldsymbol{z}$. Returns are computed using the estimated networks and firm-level knowledge capital in 2017.

investment:[7]

$$\text{Private marginal return to R\&D for firm } i \;=\; \mu\,\frac{d}{dz_i}V^i(\boldsymbol{z}) \;=\; 2\mu\left(\boldsymbol{X}_i^i\right)^\top \boldsymbol{z}$$

In vector form, this is:

$$\text{Private marginal return to R\&D} \;=\; 2\mu\,\widetilde{\boldsymbol{X}}\,\boldsymbol{z}.$$

Similarly, the social marginal return to R&D is the increase in social welfare from an additional unit of firm i's R&D:

$$\text{Social marginal return to R\&D} \;=\; \mu\,\frac{d}{d\boldsymbol{z}}V(\boldsymbol{z}) \;=\; 2\mu\,\boldsymbol{X}\,\boldsymbol{z}.$$

Figure 8 plots private and social marginal returns to R&D for each firm. The model-implied correlation between private and social marginal returns is high (0.983), but social returns to R&D tend to exceed private returns, and this tendency is more pronounced among firms with high R&D returns.

[7] In our model with quadratic R&D costs, the private marginal return to R&D, $2\mu\left(\boldsymbol{X}_i^i\right)^\top \boldsymbol{z}$, equals twice the investment level, $2x_i$.

Table 7: Growth Decomposition

Description	Growth rate in 2017
Technology Spillovers	3.86%
R&D	0.67%
Obsolescence	−3.33%
Total	1.20%

Note: Contributions to expected economic growth are decomposed using Equations (42) to (44). Calculations use the estimated ownership, technology spillover, and product-market rivalry networks, together with knowledge capital in 2017.

4.3 Growth Decomposition

In this section, we derive a formula that decomposes the expected economic growth rate into four terms: (i) technology spillovers, (ii) R&D, (iii) obsolescence (depreciation of knowledge capital), and (iv) a term related to shocks.

Using Equations (30) and (32), we apply Itô's lemma to obtain the stochastic process for log output:

$$d\log Y_t = \left[\frac{\boldsymbol{z}_t^\top\left(\boldsymbol{Q\Phi}+\boldsymbol{\Phi}^\top\boldsymbol{Q}\right)\boldsymbol{z}_t}{\boldsymbol{z}_t^\top\boldsymbol{Q}\boldsymbol{z}_t}+\gamma^2\left\{\frac{\sum_i z_{i,t}^2 Q_{ii}}{\boldsymbol{z}_t^\top\boldsymbol{Q}\boldsymbol{z}_t}-2\frac{\boldsymbol{z}_t^\top\boldsymbol{Q}\text{diag}\left(\boldsymbol{z}_t^2\right)\boldsymbol{Q}\boldsymbol{z}_t}{\left(\boldsymbol{z}_t^\top\boldsymbol{Q}\boldsymbol{z}_t\right)^2}\right\}\right]dt + 2\gamma\frac{\boldsymbol{z}_t^\top\boldsymbol{Q}\text{diag}\left(\boldsymbol{z}_t\right)}{\boldsymbol{z}_t^\top\boldsymbol{Q}\boldsymbol{z}_t}dW_t$$

(see Section A.10 for the derivation). Because the technology transition matrix $\boldsymbol{\Phi}$ consists of the exogenous technology spillover matrix $\boldsymbol{\Omega}$, the endogenous R&D term $\mu^2\widetilde{\boldsymbol{X}}$, and obsolescence $-\delta\boldsymbol{I}$, we can further decompose the first drift term:

$$\boldsymbol{Q\Phi}+\boldsymbol{\Phi}^\top\boldsymbol{Q} = (\boldsymbol{Q\Omega}+\boldsymbol{\Omega Q})+\mu^2\left(\boldsymbol{Q}\widetilde{\boldsymbol{X}}+\widetilde{\boldsymbol{X}}^\top\boldsymbol{Q}\right)-2\delta\boldsymbol{Q}$$

Given this observation, we define the contributions of technology spillovers, obsolescence, R&D, and the Itô term to the expected economic growth rate as:

$$\frac{d\log Y_t|_{\text{spillover}}}{dt} = \frac{\boldsymbol{z}_t^\top\left(\boldsymbol{Q\Omega}+\boldsymbol{\Omega Q}\right)\boldsymbol{z}_t}{\boldsymbol{z}_t^\top\boldsymbol{Q}\boldsymbol{z}_t} \tag{42}$$

$$\frac{d\log Y_t|_{\text{R\&D}}}{dt} = \mu^2\frac{\boldsymbol{z}_t^\top\left(\boldsymbol{Q}\widetilde{\boldsymbol{X}}+\widetilde{\boldsymbol{X}}^\top\boldsymbol{Q}\right)\boldsymbol{z}_t}{\boldsymbol{z}_t^\top\boldsymbol{Q}\boldsymbol{z}_t} \tag{43}$$

$$\frac{d\log Y_t|_{\text{Obsolescence}}}{dt} = -2\delta \tag{44}$$

Table 8: Counterfactual Ownership Structures

Ownership Structure	Description
Baseline	Observed ownership structure in 2017
Dispersed	$\boldsymbol{K}^{D} = \boldsymbol{I}$
Mean=1999	$\kappa_{ij,2017}^{M1999} = \text{const} \times \kappa_{ij,2017}$ and $\mathbb{E}\left[\kappa_{ij,2017}^{M1999}\right] = \mathbb{E}\left[\kappa_{ij,1999}\right]$ for $j \neq i$
Uniform	$\kappa_{ij,2017}^{U} = \mathbb{E}\left[\kappa_{lm,2017};\, l \neq m\right]$ for $j \neq i$
Monopoly	$\boldsymbol{K}^{M} = J$

$$\frac{d \log Y_t|_{\text{Itô}}}{dt} = \gamma^2 \left\{ \frac{\sum_i z_{i,t}^2 Q_{ii}}{\boldsymbol{z}_t^{\top} \boldsymbol{Q} \boldsymbol{z}_t} - 2 \frac{\boldsymbol{z}_t^{\top} \boldsymbol{Q} \operatorname{diag}\left(\boldsymbol{z}_t^2\right) \boldsymbol{Q} \boldsymbol{z}_t}{\left(\boldsymbol{z}_t^{\top} \boldsymbol{Q} \boldsymbol{z}_t\right)^2} \right\} \tag{45}$$

In Table 7, we report the results of the growth decomposition. In the baseline model, we consider a deterministic economy, so the shocks term in Equation (45) is zero. The table reports the value of each contribution—technology spillovers Equation (42), the R&D component Equation (43), and obsolescence Equation (44).

4.4 Impact of Different Ownership Structures

In this section, we investigate the impact of various counterfactual ownership structures on key aggregate variables such as total R&D investment, the expected economic growth rate, expected social welfare, and the firm value share. We compare these outcomes under five distinct ownership structures, as detailed in Table 8. The "Baseline" scenario refers to the observed ownership structure in 2017. The "Dispersed" ownership structure rules out overlapping ownership: each investor holds shares in a single firm, so the ownership matrix is the identity, $\boldsymbol{K}^{D} = \boldsymbol{I}$. This scenario represents an extreme case where corporate ownership is maximally dispersed. The "Mean=1999" scenario multiplies every off-diagonal element of the 2017 ownership matrix ($\kappa_{ij,2017}$ for $j \neq i$) by a common factor so that the resulting elements satisfy $\mathbb{E}\left[\kappa_{ij,2017}^{M1999}\right] = \mathbb{E}\left[\kappa_{ij,1999}\right]$, where $\mathbb{E}[\cdot]$ denotes the mean taken over off-diagonal entries. This counterfactual isolates the effects of the rise in common ownership during the sample period. Under the "Uniform" ownership structure, all off-diagonal elements are set to the same value equal to their 2017 average, so for $j \neq i$ we have $\kappa_{ij,2017}^{U} = \mathbb{E}\left[\kappa_{lm,2017};\, l \neq m\right]$. This structure highlights the role of heterogeneity in ownership links. Finally, the "Monopoly" structure sets $\boldsymbol{K}^{M} = J$, where J denotes the all-ones matrix, meaning each investor holds the same share in every firm and ownership is maximally concentrated.

In what follows, we focus on the case where ownership affects only R&D decisions and

Table 9: Impact of Ownership Structure on Total R&D

Total R&D in 2017 (Optimal R&D: 100)	Ownership				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Baseline	40.48	38.68	31.56	28.26	21.39
Only Business Stealing $\mathbf{\Omega} = \mathbf{0}$	52.04	49.73	41.51	36.40	30.69
Only Technology Spillovers $\mathbf{\Sigma} = \boldsymbol{I}$, $\zeta/L = 0$	13.61	14.25	18.33	19.33	27.77

Note: Values are normalized so that the social planner's optimal R&D is 100. Ownership structures: Dispersed: no common ownership; Mean (1999): common ownership at its 1999 level; Uniform: equal common-ownership weights; Baseline: the estimated 2017 ownership network; Monopoly: each owner holds the same stake in every firm. "Only Business Stealing" sets technology spillovers to zero ($\mathbf{\Omega} = \mathbf{0}$). "Only Technology Spillovers" eliminates business stealing by setting $\mathbf{\Sigma} = \boldsymbol{I}$ and $\zeta/L = 0$.

product-market competition remains dispersed, so firms maximize their own profits. The next subsection turns to the setting where ownership also shapes product-market behavior alongside R&D choices. Table 9 examines the impact on total R&D expenditure. The baseline total R&D expenditure is 28.26% of the optimal R&D level. As ownership becomes more dispersed, R&D expenditure increases: "Dispersed" achieves the highest level (40.48), followed by "Mean=1999" (38.68), "Uniform" (31.56), and "Monopoly" (21.39). Common ownership leads firms to internalize both the negative business-stealing externality and the positive technology-spillover externality. Quantitatively, the former dominates the latter.

Decomposing the counterfactuals in the second and third rows of Table 9 reveals the strategic forces behind these patterns. The second row removes technology spillovers, leaving only the business-stealing channel. In that environment, greater ownership overlap induces firms to internalize only the negative externality from business stealing, so R&D falls as ownership concentrates. The third row shuts down business stealing but keeps spillovers active; in that case, ownership concentration leads firms to internalize only the positive externality associated with technology spillovers and R&D rises.

Table 10 reports the expected economic growth rate, expected social welfare, and the firm value share across ownership structures. Social welfare is normalized to the allocation with optimal R&D under dispersed competition. Growth co-moves with total R&D: more dispersed ownership is associated with higher growth. Because the internalization of business-stealing effects dominates technology spillovers in our calibration, the growth rate declines as ownership becomes more concentrated. Social welfare also declines with concentration: R&D spending is below the social optimum under all ownership structures, and greater concentration further depresses R&D. Finally, the firm value share

Table 10: Growth, Social Welfare, and Firm Value Share by Ownership

	Ownership				
	Dispersed	Mean=1999	Uniform	Baseline	Monopoly
Economic Growth Rate (%)	1.32	1.31	1.24	1.20	1.11
Social Welfare (Optimal R&D: 100)	94.91	94.86	94.52	94.35	93.47
Firm Value Share (%)	26.63	26.72	27.20	27.24	27.82

Note: "Economic Growth Rate" reports the expected annual growth rate in percent. "Social Welfare" is consumption-equivalent welfare normalized to 100 at the allocation with planner-optimal R&D under dispersed product-market competition. Ownership structures: Dispersed: no common ownership; Mean (1999): common ownership at its 1999 level; Uniform: equal common-ownership weights; Baseline: the estimated 2017 ownership network; Monopoly: each owner holds the same stake in every firm.

Table 11: Common Ownership Affecting Only R&D vs Both R&D and Production

Production ownership structure R&D ownership structure	Dispersed Dispersed	Dispersed Common	Common Common
Total Output (Dispersed: 100)	100.00	100.00	97.26
Total R&D (Dispersed: 100)	100.00	69.81	86.36
Economic Growth Rate (%)	1.323	1.200	1.288
CE Welfare (Dispersed: 100)	100.00	99.41	97.28
Firm Value Share (%)	26.63	27.24	34.10

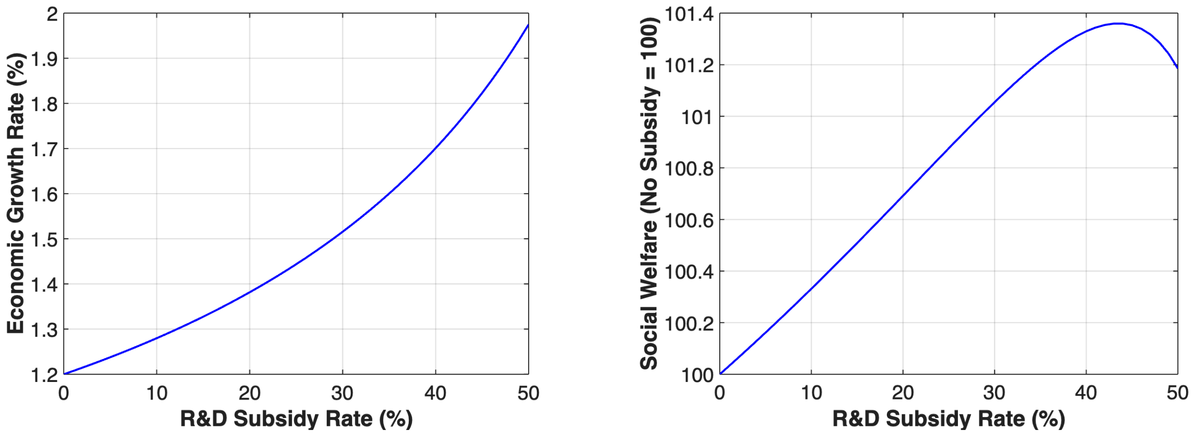
Note: "Common (R&D only)" assumes ownership affects only R&D choices while product-market competition is dispersed. "Common (Production & R&D)" assumes ownership affects both R&D and static production decisions. Values for output, R&D, and social welfare are normalized so that the dispersed-ownership case equals 100.

rises as ownership becomes more concentrated because concentration allows firm owners to capture a larger share of the gains from higher markups.

4.5 When Common Ownership Influences Only R&D Decisions

So far, we have considered settings in which the ownership structure affects only R&D investment[8]. We now compare cases in which ownership influences both product-market competition and R&D with cases in which it influences only R&D. To do so, Table 11 reports three configurations: column (1) maintains dispersed ownership for both production and R&D, column (2) introduces common ownership for R&D while keeping production dispersed, and column (3) applies common ownership to both margins.

[8]This assumption is plausible if R&D decisions reflect the preferences of higher corporate layers (e.g., boards), whereas product-market quantities reflect those of lower layers (e.g., product managers).



(a) Economic Growth Rate

(b) Social Welfare

Figure 9: Economic Growth Rate and Social Welfare under R&D Subsidy

Note: The left panel shows the expected annual growth rate across uniform R&D subsidy rates. The right panel reports consumption-equivalent social welfare, normalized so that the no-subsidy case equals 100. The subsidy rate is varied from 0 to 50% in one-percentage-point increments. Networks (ownership structure, technology spillovers, product-market rivalry) and the knowledge-capital distribution are estimated from 2017 data.

Column (1) restates the dispersed benchmark in which output, R&D, and welfare are normalized to 100, the expected growth rate is 1.323%, and the firm value share equals 26.63%. In column (2), where common ownership influences only R&D, output remains 100 but total R&D falls to 69.81 and the growth rate to 1.200%; welfare also slips to 99.41 while the firm value share rises modestly to 27.24%. These movements arise because, with production decisions still dispersed, common owners internalize the negative business-stealing externality more strongly than they value the positive technology spillovers. Column (3) allows common ownership to shape both production and R&D: output falls to 97.26, R&D rebounds partway to 86.36, and growth increases to 1.288%. The rebound in R&D occurs because common ownership makes product-market competition less aggressive, thereby raising markups and boosting the private return to innovation. Welfare drops further to 97.28, while the firm value share jumps to 34.10 as owners capture more surplus from higher markups. Overall, social welfare is lowest and the firm value share is highest when ownership influences both production and R&D.

4.6 R&D Subsidy

4.6.1 Uniform R&D Subsidy

We first analyze the optimal uniform R&D subsidy. A subsidy rate s reduces each firm's private R&D cost by the factor $1 - s$, so the firm's R&D policy satisfies $\boldsymbol{x} = (1-s)^{-1}\mu\widetilde{\boldsymbol{X}}\boldsymbol{z}$. In the welfare calculation, however, R&D uses real resources, so household consumption subtracts the full resource cost $\boldsymbol{x}^{\top}\boldsymbol{x}$. The subsidy is therefore treated as a distortion to private R&D incentives financed by lump-sum transfers, not as a reduction in the economy's resource cost of innovation.

The uniform subsidy does not alter the static production rule conditional on the knowledge-capital vector $\boldsymbol{z}_t$. Its effect on welfare works only through the induced dynamic path of knowledge capital. Therefore, welfare can improve only to the extent that the subsidy closes the gap between equilibrium R&D and the constrained planner's R&D allocation, while avoiding excessive R&D at high subsidy rates. We solve the equilibrium over a grid of subsidy rates from 0 to 50% in one-percentage-point increments. Welfare reaches its maximum at a subsidy of 43% (Figure 9). At that rate, the expected growth rate increases to 1.77%—0.57 percentage points above the no-subsidy equilibrium under baseline common ownership (Figure 9a)—and consumption-equivalent welfare improves by 1.36% (Figure 9b). The gain is nonetheless far smaller than the 5.99% increase delivered by the constrained social planner's optimal R&D level under baseline common ownership weights. Large welfare gains therefore require policies that exploit network heterogeneity rather than uniform subsidies alone.

4.6.2 Targeted R&D Subsidies

To study how firms choose equilibrium R&D effort and determine which firms should receive targeted R&D subsidies, we regress each firm's equilibrium R&D and the ratio of the social to private value of R&D on observable firm characteristics. The sample combines parameter values identified from 2017 data with the implied equilibrium outcomes generated by the model for that year.

Column 1 of Table 12 reports regressions of equilibrium R&D on initial knowledge capital and on eigenvector centrality in networks constructed from common ownership weights, product proximity, and technology proximity. The estimates indicate that firms with greater initial knowledge capital choose higher R&D, reflecting scale effects in innovation. Firms that are more central in the product-market network invest less in R&D because other firms supply similar products, which compresses the private return from their R&D. By contrast, firms with higher technology-spillover centrality tend to invest

Table 12: Determinants of Private and Social Returns to R&D

	Private R&D x	Social / Private value of R&D
Initial knowledge capital z	0.122***	-0.000212
	(0.000848)	(0.000158)
Product market centrality	-16.4***	-1.06***
	(0.286)	(0.0532)
Technology spillover centrality	5.45***	1.48***
	(0.423)	(0.0788)
Ownership structure centrality	-7.16***	0.580***
	(0.239)	(0.0446)
Intercept	-22.4***	0.900***
	(0.255)	(0.0475)
Observations	740	740
R^2	0.976	0.555

Note: Coefficients are from firm-level regressions with robust standard errors in parentheses. The sample combines parameter values identified from 2017 data with the implied equilibrium outcomes generated by the model for that year. The dependent variable is private R&D in column 1 and the ratio of social to private value in column 2. Initial knowledge capital is the firm's calibrated knowledge stock. Product-market, technology-spillover, and ownership-structure centrality are eigenvector centralities computed on the corresponding network matrices. *** indicates significance at the 1% level.

more in R&D, as spillovers raise expected future growth and private returns. Greater centrality in the ownership structure is associated with lower R&D because firms with substantial overlapping ownership internalize the business-stealing externality more than the technology spillovers, dampening their R&D incentives.

Column 2 of Table 12 uses the ratio of social to private R&D value as the dependent variable. A higher ratio therefore indicates that the firm underinvests in R&D relative to the social optimum and should receive a larger subsidy. Initial knowledge capital has no statistically significant effect on the ratio. Product proximity centrality has a negative and significant coefficient. Firms that face many close competitors in the product market create large negative externalities of business stealing, so they should receive smaller R&D subsidies. Technology proximity centrality has a positive and significant coefficient, which points to strong technology spillovers and a need for larger subsidies. Ownership-structure centrality is also significant and positive. Because the regression already conditions on product and technology centrality, targeted subsidies must offset the portion of the externalities that firms have already internalized.

5 Alternative Models of Corporate Governance

We examine various assumptions about corporate governance and compute the resulting economic growth rates and social welfare under each assumption. Specifically, following Ederer and Pellegrino (2024), we focus on three models: (1) super-proportional influence of large investors, (2) blockholder thresholds, and (3) governance frictions and managerial entrenchment. By comparing these scenarios, we analyze the robustness of our findings.

5.1 Super-Proportional Influence of Large Investors

Following Gilje et al. (2020) and Backus et al. (2021), we assume that firm i assigns weights $\gamma_{io} s_{io}$ to the profits of investor o, where γ_{io} captures how a large investor may exert greater influence than smaller ones. Then, the influence-adjusted common-ownership weight $\tilde{\kappa}_{ij}$ is

$$\tilde{\kappa}_{ij} = \frac{\sum_{o=1}^{O} \gamma_{io}\, s_{io}\, s_{jo}}{\sum_{o=1}^{O} \gamma_{io}\, s_{io}^2}, \quad i \neq j.$$

Setting $\gamma_{io} = 1$ nests the baseline. Following Ederer and Pellegrino (2024), we set $\gamma_{io} = \sqrt{s_{io}}$, which amplifies the impact of large shareholders on firm decisions.

5.2 Blockholder Thresholds

An alternative way to model the disproportionate control of large investors is to assume that only investors who own at least a given fraction of a firm's equity actively influence that firm's decisions. Accordingly, firm i internalizes investor o's portfolio effects only if o is a blockholder of firm i. We set the threshold to 5% of the firm's outstanding shares. Define $b_{io} \equiv \mathbb{1}\{s_{io} \geq 0.05\}$. The resulting common-ownership weight is

$$\tilde{\kappa}_{ij} = \frac{\sum_{o=1}^{O} b_{io}\, s_{io}\, s_{jo}}{\sum_{o=1}^{O} b_{io}\, s_{io}^2}, \quad i \neq j.$$

5.3 Governance Frictions and Managerial Entrenchment

Corporate managers may not fully maximize shareholder value due to governance frictions and entrenchment. Azar and Ribeiro (2021) construct a structural model of industrial organization and estimate it using airline industry data to quantify common ownership weights. They propose a flexible objective function in which the manager of firm i discounts

Table 13: Alternative Models of Corporate Governance

	Ownership Structure in 2017					
	Dispersed Ownership	Baseline: Proportional Influence	Super Proportional Influence	Blockholder Influence	Governance Frictions (Uniform)	Governance Frictions (Firm-Specific)
Total R&D Expenditure	100.00	69.81	68.97	77.45	90.32	90.41
Expected Growth Rate (%)	1.323	1.200	1.194	1.234	1.287	1.289
Expected Social Welfare	100.00	99.41	99.37	99.59	99.86	99.86
Firm Value Share (%)	26.63	27.24	27.24	27.09	26.82	26.84

Note: This table shows the results of the alternative models of corporate governance. The baseline model is the proportional influence model, where the firm assigns weights to the profits of investors proportional to their ownership shares. The super-proportional influence model assumes that large investors have more influence than smaller ones. The blockholder influence model assumes that only blockholders influence the firm's decisions. The governance frictions model assumes that corporate managers may not fully maximize shareholder value due to governance frictions and entrenchment. The uniform governance frictions model assumes that the mitigation factor is uniform across firms, while the firm-specific governance frictions model assumes that the mitigation factor varies across firms and over time. R&D expenditure and expected social welfare under a dispersed ownership scenario are normalized to 100.

the profits of other firms by τ_i compared to the case of proportional influence:

$$\pi_i + \tau_i \sum_{j \neq i} \kappa_{ij} \pi_j.$$

We analyze two specifications highlighted by Azar and Ribeiro (2021). First, we estimate our model using a constant mitigation factor, $\tau = 0.29$, which is uniform across firms. Second, we employ the best-fitting structural specification in Azar and Ribeiro (2021), where the mitigation factor varies across firms and over time, defined as:

$$\tau_{i,t} = \frac{\exp\left[\theta_0 + \log\left(\mathrm{IHHI}_{i,t}\right)\right]}{1 + \exp\left[\theta_0 + \log\left(\mathrm{IHHI}_{i,t}\right)\right]}$$

with $\theta_0 = 2.6844$, where $\mathrm{IHHI}_{i,t} = \sum_{o=1}^{O} s_{io,t}^2$ is the investor Herfindahl concentration index.

5.4 Results

Table Table 13 presents comparative statics of how different corporate governance models in the 2017 ownership structure affect key variables: R&D expenditure, expected growth rate, expected social welfare, and firm value share. For R&D expenditure and expected social welfare, values under the dispersed-ownership scenario are normalized to 100.

Relative to the dispersed-ownership benchmark, the baseline proportional-influence model lowers R&D expenditure to 69.81, the expected growth rate to 1.200, and social welfare to 99.41. Super-proportional influence amplifies these effects (R&D 68.97, growth 1.194, welfare 99.37). A blockholder threshold attenuates them (R&D 77.45, growth 1.234, welfare 99.59). Introducing governance frictions substantially mitigates the impact: with a uniform mitigation factor, R&D is 90.32, growth 1.287, and welfare 99.86; with a firm-specific factor, R&D is 90.41, growth 1.289, and welfare 99.86. Although the quantitative impact differs across assumptions, common ownership has the same qualitative impact under all corporate governance assumptions. Importantly, across all robustness checks, firms still internalize rivals' business-stealing losses more strongly than technology spillovers, so the net effect of common ownership remains negative for R&D and growth.

6 Conclusion

We develop an endogenous growth model that incorporates three inter-firm networks: ownership structure, product-market rivalry, and technological spillovers. Using this framework, we examine how ownership structures affect economic growth and social welfare. To quantify these effects, we estimate the model using data on more than 700 publicly traded U.S. firms with patent holdings and compute the dynamic oligopoly equilibrium. Our numerical analysis suggests that in 2017 the observed level of common ownership reduced the expected economic growth rate by 0.12 percentage points and social welfare by 0.59% relative to a counterfactual with fully dispersed ownership. This pattern implies that, under common ownership, the internalization of business-stealing externalities dominates the internalization of technological spillovers.

While we apply the framework to U.S. common ownership, it can be applied to other settings. In East and Southeast Asian economies such as South Korea, family-controlled conglomerates (chaebols) account for a large share of activity. In Japan, conglomerates called zaibatsu were dissolved by the Allied occupation forces at the end of World War II. Our framework can be used to assess how such large conglomerates (and their dissolution) affect product-market competition, R&D activity, and economic growth.

The framework also allows us to analyze foreign direct investment (FDI) in developing economies, where policymakers balance market-share losses to foreign entrants against gains from technological spillovers. Our approach provides a quantitative assessment of this net effect.

Finally, we can analyze technology licensing by treating licenses as claims on the product-market profits generated by the licensed technology, which then enter the framework analogously to ownership shares. Quantifying how licensing shapes product-market competition, R&D investment, and aggregate growth is a promising direction for future research.

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Appendix

Appendix A Theoretical Appendix

A.1 Derivation of Static Equilibrium

Inserting the optimality condition for technology choice (20) and (21) into the optimality condition for quantity choice (19), we obtain

$$z_{i,t} = 2\sqrt{\zeta \frac{w_t}{P_t}} + \sum_j \left(\sigma_{ij} + \sigma_{ij}\kappa_{ij}\right) q_{j,t}$$

Inserting the expression for real wage (22), we obtain

$$z_{i,t} = \sum_j \left(2\frac{\zeta}{L} + \sigma_{ij} + \sigma_{ij}\kappa_{ij}\right) q_{j,t}.$$

In vector form,

$$\boldsymbol{z}_t = \left(2\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma}\right) \boldsymbol{q}_t.$$

Therefore, the quantities are expressed as linear functions of knowledge capital:

$$\boldsymbol{q}_t = \boldsymbol{N} \boldsymbol{z}_t$$

where

$$\boldsymbol{N} \equiv \left(2\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma}\right)^{-1}.$$

A.2 Derivation of Quadratic Form of Static Objective

Substituting the FOC with respect to q_i into the expression for the profit (18), we obtain

$$\pi_{i,t} = \sum_j \kappa_{ij}\sigma_{ij} q_{i,t} q_{j,t}$$

Therefore, the static objective function of firm i is rewritten as:

$$\sum_j \kappa_{ij}\pi_{j,t} = \sum_h \kappa_{ih} \sum_j \kappa_{hj}\sigma_{hj}q_{h,t}q_{j,t} \tag{46}$$

From equation (23), we obtain

$$q_{h,t}q_{j,t} = \boldsymbol{z}_t^\top \left(\frac{\boldsymbol{N}_h^\top \boldsymbol{N}_j + \boldsymbol{N}_j^\top \boldsymbol{N}_h}{2} \right) \boldsymbol{z}_t$$

Inserting the above equation into the firm's static objective function (46), we express the objective in a quadratic form of the firm's productivity vector $\boldsymbol{z}_t$ (here, $\boldsymbol{N}_i$ denotes the i-th row of $\boldsymbol{N}$):

$$\sum_j \kappa_{ij}\pi_{j,t} = \boldsymbol{z}_t^\top \boldsymbol{Q}^i \boldsymbol{z}_t$$

where

$$\boldsymbol{Q}^i \equiv \sum_h \kappa_{ih} \sum_j \kappa_{hj}\sigma_{hj} \frac{\boldsymbol{N}_h^\top \boldsymbol{N}_j + \boldsymbol{N}_j^\top \boldsymbol{N}_h}{2}$$

A.3 Derivation of Output

Let $\mathbf{1}$ denote an $n \times 1$ vector with all elements equal to 1. Rewrite (21) in vector form:

$$\boldsymbol{b}_t = \boldsymbol{z}_t - \sqrt{\zeta \frac{w_t}{P_t}}\mathbf{1}$$

Inserting (22), we obtain

$$\begin{aligned} \boldsymbol{b}_t &= \boldsymbol{z}_t - \frac{\zeta}{L}\left(\sum_i q_{i,t}\right)\mathbf{1} \\ &= \boldsymbol{z}_t - \frac{\zeta}{L}\boldsymbol{J}\boldsymbol{q}_t. \end{aligned}$$

From (23), we obtain

$$\boldsymbol{b}_t = \left(\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma}\right)\boldsymbol{q}_t$$

Therefore, from (4), the total output is given by

$$
\begin{aligned}
Y_t &= \boldsymbol{q}_t^\top \left(\frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma} \right) \boldsymbol{q}_t - \frac{1}{2} \boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t \\
&= \boldsymbol{q}_t^\top \left(\frac{\zeta}{L} \boldsymbol{J} + \frac{1}{2} \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma} \right) \boldsymbol{q}_t \\
&= \boldsymbol{z}_t^\top \boldsymbol{N}^\top \left(\frac{\zeta}{L} \boldsymbol{J} + \frac{1}{2} \boldsymbol{\Sigma} + \boldsymbol{K} \circ \boldsymbol{\Sigma} \right) \boldsymbol{N} \boldsymbol{z}_t \\
&= \frac{1}{2} \boldsymbol{z}_t^\top \boldsymbol{N}^\top \left(2 \frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{\Sigma} \circ \left(\boldsymbol{K} + \boldsymbol{K}^\top \right) \right) \boldsymbol{N} \boldsymbol{z}_t \\
&= \boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t
\end{aligned}
$$

where

$$
\boldsymbol{Q} \equiv \frac{1}{2} \boldsymbol{N}^\top \left(2 \frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} + \boldsymbol{\Sigma} \circ \left(\boldsymbol{K} + \boldsymbol{K}^\top \right) \right) \boldsymbol{N}
$$

A.4 Derivation of Dynamic Riccati and Lyapunov Equations

This subsection derives the equations used to solve the dynamic R&D game and to evaluate welfare after the equilibrium policy is obtained. Guess that firm i's value function is quadratic,

$$
V^i(\boldsymbol{z}) = \boldsymbol{z}^\top \boldsymbol{X}^i \boldsymbol{z},
$$

with $\boldsymbol{X}^i$ symmetric. The first-order condition with respect to x_i in Equation (24) gives

$$
x_i = \mu \left(\boldsymbol{X}_i^i \right)^\top \boldsymbol{z},
$$

where $\boldsymbol{X}_i^i$ is the i-th column of $\boldsymbol{X}^i$. Stacking these policy coefficients yields $\boldsymbol{x} = \mu \widetilde{\boldsymbol{X}} \boldsymbol{z}$, where

$$
\widetilde{\boldsymbol{X}} \equiv \left[\begin{array}{ccc} \boldsymbol{X}_1^1 & \cdots & \boldsymbol{X}_n^n \end{array} \right]^\top .
$$

Thus the drift of knowledge capital under the policy profile is

$$
\boldsymbol{\Phi} \boldsymbol{z} = \left(\boldsymbol{\Omega} - \delta \boldsymbol{I} + \mu^2 \widetilde{\boldsymbol{X}} \right) \boldsymbol{z}.
$$

Substituting the quadratic guess and the policy profile into firm i's HJB gives

$$\rho\, \boldsymbol{z}^\top \boldsymbol{X}^i \boldsymbol{z} = \boldsymbol{z}^\top \boldsymbol{Q}^i \boldsymbol{z} - \mu^2 \sum_j \kappa_{ij} \boldsymbol{z}^\top \boldsymbol{X}_j^j \left(\boldsymbol{X}_j^j\right)^\top \boldsymbol{z} + \boldsymbol{z}^\top \left(\left(\boldsymbol{\Phi}^\top \boldsymbol{X}^i + \boldsymbol{X}^i \boldsymbol{\Phi}\right) + \gamma^2 \boldsymbol{X}^i\right) \boldsymbol{z}.$$

Since this equality must hold for all $\boldsymbol{z}$, firm i's matrix solves

$$0 = \boldsymbol{Q}^i - \mu^2 \sum_j \kappa_{ij} \boldsymbol{X}_j^j \left(\boldsymbol{X}_j^j\right)^\top + \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}\right)^\top \boldsymbol{X}^i + \boldsymbol{X}^i \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}\right).$$

These are the stacked algebraic Riccati equations in Equation (31).

Once the equilibrium policy $\boldsymbol{x} = \mu \widetilde{\boldsymbol{X}} \boldsymbol{z}$ has been solved, household welfare is a policy-evaluation problem rather than an optimization problem. Let

$$V\left(\boldsymbol{z}\right) = \boldsymbol{z}^\top \boldsymbol{X} \boldsymbol{z}.$$

Using $Y = \boldsymbol{z}^\top \boldsymbol{Q} \boldsymbol{z}$ and $\boldsymbol{x}^\top \boldsymbol{x} = \boldsymbol{z}^\top \mu^2 \widetilde{\boldsymbol{X}}^\top \widetilde{\boldsymbol{X}} \boldsymbol{z}$, the household HJB becomes

$$\rho V\left(\boldsymbol{z}\right) = \boldsymbol{z}^\top \left(\boldsymbol{Q} - \mu^2 \widetilde{\boldsymbol{X}}^\top \widetilde{\boldsymbol{X}}\right) \boldsymbol{z} + V_z\left(\boldsymbol{z}\right) \boldsymbol{\Phi} \boldsymbol{z} + \frac{\gamma^2}{2} \boldsymbol{z}^\top V_{zz}\left(\boldsymbol{z}\right) \boldsymbol{z}.$$

Substituting the quadratic form gives the Lyapunov equation

$$0 = \boldsymbol{Q} - \mu^2 \widetilde{\boldsymbol{X}}^\top \widetilde{\boldsymbol{X}} + \boldsymbol{X} \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}\right) + \left(\boldsymbol{\Phi} - \frac{1}{2}\left(\rho - \gamma^2\right) \boldsymbol{I}\right)^\top \boldsymbol{X}.$$

The total value of firms is obtained from the same policy-evaluation equation by replacing $\boldsymbol{Q}$ with the producer-surplus matrix $\boldsymbol{P}$.

A.5 Derivation of the Solution of Social Planner's Static Problem

Define the Lagrangian as

$$\mathcal{L} = \boldsymbol{q}_t^\top \left(z_t - \zeta \boldsymbol{a}_t\right) - \frac{1}{2} \boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t - \lambda_t \left\{\sum_i \frac{q_{i,t}}{a_{i,t}} - L\right\}$$

The FOC w.r.t. $a_{i,t}$ gives

$$a_{i,t} = \sqrt{\frac{\lambda_t}{\zeta}} \tag{47}$$

and therefore,

$$b_{i,t} = z_{i,t} - \sqrt{\zeta \lambda_t} \tag{48}$$

Inserting (47) into (34), we obtain

$$\sqrt{\zeta \lambda_t} = \frac{\zeta}{L} \sum_i q_{i,t} \tag{49}$$

Note that

$$\sqrt{\zeta \lambda_t} \mathbf{1} = \frac{\zeta}{L} \left(\sum_i q_{i,t} \right) \mathbf{1} = \frac{\zeta}{L} \boldsymbol{J} \boldsymbol{q}_t. \tag{50}$$

The FOC w.r.t. $\boldsymbol{q}_t$ gives

$$\boldsymbol{z}_t - \zeta \boldsymbol{a}_t - \boldsymbol{\Sigma} \boldsymbol{q}_t = \lambda_t \begin{bmatrix} 1/a_{1,t} \\ 1/a_{2,t} \\ \vdots \\ 1/a_{n,t} \end{bmatrix} \tag{51}$$

Inserting (47), we obtain

$$\boldsymbol{z}_t = 2\sqrt{\zeta \lambda_t} \mathbf{1} + \boldsymbol{\Sigma} \boldsymbol{q}_t$$

Inserting (50), we obtain

$$\boldsymbol{z}_t = \left(2 \frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} \right) \boldsymbol{q}_t$$

Therefore,

$$\boldsymbol{q}_t = \boldsymbol{N}^* \boldsymbol{z}_t \tag{52}$$

where

$$\boldsymbol{N}^* = \left(2 \frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} \right)^{-1}$$

To obtain the quadratic form of output, inserting (48) into (4),

$$Y_t = \boldsymbol{q}_t^\top \left(\boldsymbol{z}_t - \sqrt{\zeta \lambda_t} \mathbf{1} \right) - \frac{1}{2} \boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t$$

Inserting (50) and using (52),

$$\begin{aligned} Y_t &= \boldsymbol{q}_t^\top \left(\left(2 \frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} \right) \boldsymbol{q}_t - \frac{\zeta}{L} \boldsymbol{J} \boldsymbol{q}_t \right) - \frac{1}{2} \boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t \\ &= \boldsymbol{q}_t^\top \left(\frac{\zeta}{L} \boldsymbol{J} + \frac{1}{2} \boldsymbol{\Sigma} \right) \boldsymbol{q}_t \end{aligned}$$

$$= \frac{1}{2} \boldsymbol{q}_t^\top \boldsymbol{N}^{*-1} \boldsymbol{q}_t$$
$$= \frac{1}{2} \boldsymbol{z}_t^\top \boldsymbol{N}^{*} \boldsymbol{z}_t$$

A.6 Monopoly

As the third counterfactual scenario, we consider the "monopoly" case, where all firms are owned by a single investor, to analyze the allocation of R&D, its impact on economic growth, and social welfare under an extremely concentrated ownership structure. Alternatively, in our setting, where firms maximize the weighted sum of investors' returns, the monopoly case is equivalent to a situation where multiple investors exist but all hold identical portfolios of firm ownership. This corresponds to a common ownership matrix where all elements are equal to 1. In the monopoly case, since all firms share the same objective function, the equilibrium can be determined by solving a single Riccati equation.

Static Problem

First, we characterize the static allocation in the monopolist owner's problem. Like our baseline static Cournot-Nash equilibrium, we assume that the monopolist takes real wage w_t/P_t as given in the static problem. The monopolist chooses $\boldsymbol{a}_t$, $\boldsymbol{b}_t$, and $\boldsymbol{q}_t$ to maximize the producer surplus Π_t, which is defined as

$$\begin{aligned} \Pi_t &\equiv \frac{\boldsymbol{q}_t^\top \boldsymbol{p}_t}{P_t} - \frac{w_t}{P_t} \sum_i l_{i,t} \\ &= \boldsymbol{q}_t^\top \boldsymbol{b}_t - \boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t - \frac{w_t}{P_t} \sum_i \frac{q_{i,t}}{a_{i,t}} \end{aligned}$$

subject to the technology choice constraint $z_{i,t} = \zeta a_{i,t} + b_{i,t}$ for each firm. By solving the monopolist's static problem, we obtain a solution of quantities as a linear function of knowledge capital:

$$\boldsymbol{q}_t^M = \boldsymbol{N}^M \boldsymbol{z}_t$$

where

$$\boldsymbol{N}^M \equiv \frac{1}{2} \left(\frac{\zeta}{L} \boldsymbol{J} + \boldsymbol{\Sigma} \right)^{-1}.$$

and a quadratic form for the total profit of knowledge capital:

$$\Pi_t^M = \boldsymbol{z}_t^\top \boldsymbol{P}^M \boldsymbol{z}_t$$

where

$$\boldsymbol{P}^M = \left(\boldsymbol{N}^M\right)^\top \boldsymbol{\Sigma} \boldsymbol{N}^M$$

(The derivation is in Section A.7.)

Dynamic Problem

Next, we characterize the dynamic allocation in the monopolist owner's problem. The monopolist's dynamic problem is formulated as follows:

$$V^{M,P}(\boldsymbol{z}_0) \equiv \max_{\{\boldsymbol{x}_t\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^\infty \exp(-\rho t) \left\{ \boldsymbol{z}_t^\top \boldsymbol{P}^M \boldsymbol{z}_t - \boldsymbol{x}_t^\top \boldsymbol{x}_t \right\} dt \right],$$

subject to

$$d\boldsymbol{z}_t = \left((\boldsymbol{\Omega} - \delta \boldsymbol{I}) \boldsymbol{z}_t + \mu \boldsymbol{x}_t \right) dt + \gamma \, \mathrm{diag}(\boldsymbol{z}_t) \, d\boldsymbol{W}_t$$

We obtain the monopoly allocation by solving the modified version of the Riccati equation (38), in which $\frac{1}{2}\boldsymbol{N}^*$ is replaced with $\boldsymbol{P}^M$. Denote the solution to this producer-surplus problem by $\boldsymbol{X}^{M,P}$. Then the R&D allocation is

$$\boldsymbol{x}_t = \mu \boldsymbol{X}^{M,P} \boldsymbol{z}_t$$

Welfare

In contrast to the social planner's problem, the monopolist maximizes producer surplus, which is distinct from household utility. Therefore, calculating household utility necessitates a further step. The total output is given by

$$Y_t^M = \boldsymbol{z}_t^\top \boldsymbol{Q}^M \boldsymbol{z}_t$$

where

$$\boldsymbol{Q}^M \equiv \frac{1}{2} \left(\boldsymbol{N}^M\right)^\top \left(2\frac{\zeta}{L} \boldsymbol{J} + 3\boldsymbol{\Sigma} \right) \boldsymbol{N}^M$$

(The derivation is in Section A.8.) Let $V^{M,W}$ denote the value of the representative household under the monopoly allocation, defined as

$$V^{M,W}\left(z_0\right)=\mathbb{E}_0\left[\int_0^{\infty}\exp\left(-\rho t\right)\left(\boldsymbol{z}_t^{\top}\boldsymbol{Q}^M\boldsymbol{z}_t-\boldsymbol{x}_t^{\top}\boldsymbol{x}_t\right)dt\middle|\boldsymbol{z}_0\right]$$

subject to the law of motion:

$$d\boldsymbol{z}_t=\boldsymbol{\Phi}^M\boldsymbol{z}_t dt+\gamma\,\mathrm{diag}\left(\boldsymbol{z}_t\right)d\boldsymbol{W}_t$$

where the technology transition under monopoly allocation $\boldsymbol{\Phi}^M$ is given by

$$\boldsymbol{\Phi}^M\equiv\left(\boldsymbol{\Omega}-\delta\boldsymbol{I}\right)+\mu^2\boldsymbol{X}^{M,P}.$$

The monopoly policy is already fixed at this point. Therefore, household welfare under monopoly is obtained from a Lyapunov equation, not from another Riccati equation. Guess $V^{M,W}\left(\boldsymbol{z}\right)=\boldsymbol{z}^{\top}\boldsymbol{X}^{M,W}\boldsymbol{z}$. Since $\boldsymbol{x}^{\top}\boldsymbol{x}=\boldsymbol{z}^{\top}\mu^2\left(\boldsymbol{X}^{M,P}\right)^{\top}\boldsymbol{X}^{M,P}\boldsymbol{z}$, policy evaluation gives

$$0=\boldsymbol{Q}^M-\mu^2\left(\boldsymbol{X}^{M,P}\right)^{\top}\boldsymbol{X}^{M,P}+\boldsymbol{X}^{M,W}\left(\boldsymbol{\Phi}^M-\frac{1}{2}\left(\rho-\gamma^2\right)\boldsymbol{I}\right)+\left(\boldsymbol{\Phi}^M-\frac{1}{2}\left(\rho-\gamma^2\right)\boldsymbol{I}\right)^{\top}\boldsymbol{X}^{M,W}. \tag{53}$$

Therefore, $\boldsymbol{X}^{M,W}$ is obtained as the solution of the Lyapunov equation (53), and household welfare under monopoly is $V^{M,W}\left(\boldsymbol{z}_0\right)=\boldsymbol{z}_0^{\top}\boldsymbol{X}^{M,W}\boldsymbol{z}_0$.

A.7 Derivation of the Static Solution of Monopolist's Static Problem

The FOC w.r.t. $q_{i,t}$ gives

$$0=b_{i,t}-\frac{1}{a_{i,t}}\frac{w_t}{P_t}-2\sum_j\sigma_{ij}q_{j,t} \tag{54}$$

Therefore, the real profit of each firm is given by:

$$\begin{aligned}\pi_{i,t}&=\left(b_{i,t}-\frac{1}{a_{i,t}}\frac{w_t}{P_t}\right)q_{i,t}-\sum_j\sigma_{ij}q_{i,t}q_{j,t}\\&=2\sum_j\sigma_{ij}q_{j,t}q_{i,t}-\sum_j\sigma_{ij}q_{i,t}q_{j,t}\\&=\sum_j\sigma_{ij}q_{i,t}q_{j,t}\end{aligned} \tag{55}$$

The FOC w.r.t. $a_{i,t}$ gives

$$a_{i,t} = \sqrt{\frac{1}{\zeta}\frac{w_t}{P_t}}, \tag{56}$$

and therefore,

$$b_{i,t} = z_{i,t} - \sqrt{\zeta \frac{w_t}{P_t}} \tag{57}$$

Inserting (56) into the labor market clearing condition (14), we obtain

$$\sqrt{\zeta \frac{w_t}{P_t}} = \frac{\zeta}{L}\sum_i q_{i,t} \tag{58}$$

Inserting the optimality condition for technology choice (56) and (57) into the optimality condition for quantity choice (54), we obtain

$$z_{i,t} = 2\sqrt{\frac{\zeta w_t}{P_t}} + 2\sum_j \sigma_{ij} q_{j,t}.$$

Inserting the expression for real wage (58), we obtain

$$z_{i,t} = \sum_j \left(2\frac{\zeta}{L} + 2\sigma_{ij}\right) q_{j,t}.$$

In vector form,

$$\boldsymbol{z}_t = 2\left(\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma}\right)\boldsymbol{q}_t. \tag{59}$$

Therefore, the quantities are expressed as linear functions of knowledge capital:

$$\boldsymbol{q}_t^M = \boldsymbol{N}^M \boldsymbol{z}_t \tag{60}$$

where

$$\boldsymbol{N}^M \equiv \frac{1}{2}\left(\frac{\zeta}{L}\boldsymbol{J} + \boldsymbol{\Sigma}\right)^{-1}.$$

From (55), the total profit is given by

$$\Pi_t = \boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t$$

Inserting (60) , we obtain

$$\Pi_t = \boldsymbol{z}_t^\top \left(\boldsymbol{N}^M\right)^\top \boldsymbol{\Sigma} \boldsymbol{N}^M \boldsymbol{z}_t$$

A.8 Derivation of Output in Monopolist's Problem

Similar to competitive equilibrium, we have

$$\boldsymbol{b}_t = \boldsymbol{z}_t - \frac{\zeta}{L} \boldsymbol{J} \boldsymbol{q}_t.$$

From (59), we obtain

$$\boldsymbol{b}_t = \left(\frac{\zeta}{L}\boldsymbol{J} + 2\boldsymbol{\Sigma}\right) \boldsymbol{q}_t$$

Therefore, from (4), the total output is given by

$$\begin{aligned} Y_t &= \boldsymbol{q}_t^\top \left(\frac{\zeta}{L}\boldsymbol{J} + 2\boldsymbol{\Sigma}\right) \boldsymbol{q}_t - \frac{1}{2}\boldsymbol{q}_t^\top \boldsymbol{\Sigma} \boldsymbol{q}_t \\ &= \frac{1}{2}\boldsymbol{q}_t^\top \left(2\frac{\zeta}{L}\boldsymbol{J} + 3\boldsymbol{\Sigma}\right) \boldsymbol{q}_t \\ &= \boldsymbol{z}_t^\top \boldsymbol{Q}^M \boldsymbol{z}_t \end{aligned}$$

where

$$\boldsymbol{Q}^M = \frac{1}{2}\left(\boldsymbol{N}^M\right)^\top \left(2\frac{\zeta}{L}\boldsymbol{J} + 3\boldsymbol{\Sigma}\right) \boldsymbol{N}^M$$

A.9 Interior Linear Solution and Nonnegativity

The closed-form allocations above characterize the interior linear equilibrium. That is, the formulas assume that the implied quantities and R&D efforts are nonnegative and that the relevant static matrices are nonsingular. These conditions are checked in the numerical implementation. If a nonnegativity constraint binds in the static problem, the same first-order conditions apply on the active set of firms with positive output, with the corresponding rows and columns of the static matrices restricted to that active set and quantities for inactive firms set to zero. Similarly, if an R&D effort is constrained at zero, the linear policy is replaced by the Kuhn–Tucker condition for that firm. The baseline quantitative exercises use the interior linear solution, which preserves the tractability of the Riccati and Lyapunov systems.

A.10 Derivation of Stochastic Process of Output

Let $d\boldsymbol{z}_t = \boldsymbol{\mu}_t dt + \boldsymbol{\Gamma}_t d\boldsymbol{W}_t$. Ito's lemma states that

$$df(\boldsymbol{z}_t) = \left\{ \left(f_z(\boldsymbol{z}_t) \right)^\top \boldsymbol{\mu}_t + \frac{1}{2} \mathrm{Tr} \left[\boldsymbol{\Gamma}_t^\top f_{zz}(\boldsymbol{z}_t) \boldsymbol{\Gamma}_t \right] \right\} dt + \left(f_z(\boldsymbol{z}_t) \right)^\top \boldsymbol{\Gamma}_t d\boldsymbol{W}_t$$

We apply this lemma to $d\boldsymbol{z}_t = \boldsymbol{\Phi} \boldsymbol{z}_t dt + \gamma \mathrm{diag}(\boldsymbol{z}_t) d\boldsymbol{W}_t$ and $f(\boldsymbol{z}_t) = \boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t = Y_t$. Note that

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\Phi} \boldsymbol{z}_t \\ \boldsymbol{\Gamma}_t &= \gamma \mathrm{diag}(\boldsymbol{z}_t) \\ f_z(\boldsymbol{z}_t) &= 2\boldsymbol{Q} \boldsymbol{z}_t \\ f_{zz}(\boldsymbol{z}_t) &= 2\boldsymbol{Q} \end{aligned}$$

Therefore,

$$\begin{aligned} dY_t &= \left\{ 2\boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{\Phi} \boldsymbol{z}_t + \frac{1}{2} \mathrm{Tr} \left[\gamma^2 \mathrm{diag}(\boldsymbol{z}_t)(2\boldsymbol{Q}) \mathrm{diag}(\boldsymbol{z}_t) \right] \right\} dt + 2\gamma \boldsymbol{z}_t^\top \boldsymbol{Q} \mathrm{diag}(\boldsymbol{z}_t) d\boldsymbol{W}_t \\ &= \left\{ \boldsymbol{z}_t^\top \left(\boldsymbol{Q}\boldsymbol{\Phi} + \boldsymbol{\Phi}^\top \boldsymbol{Q} \right) \boldsymbol{z}_t + \gamma^2 \sum_i z_{i,t}^2 Q_{ii} \right\} dt + 2\gamma \boldsymbol{z}_t^\top \boldsymbol{Q} \mathrm{diag}(\boldsymbol{z}_t) d\boldsymbol{W}_t \end{aligned}$$

Next, applying Ito's lemma to $\log Y_t = g(Y_t)$,

$$\begin{aligned} d \log Y_t = &\left[\frac{\boldsymbol{z}_t^\top \left(\boldsymbol{Q}\boldsymbol{\Phi} + \boldsymbol{\Phi}^\top \boldsymbol{Q} \right) \boldsymbol{z}_t}{\boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t} + \gamma^2 \left\{ \frac{\sum_i z_{i,t}^2 Q_{ii}}{\boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t} - 2 \frac{\boldsymbol{z}_t^\top \boldsymbol{Q} \mathrm{diag}\left(\boldsymbol{z}_t^2 \right) \boldsymbol{Q} \boldsymbol{z}_t}{\left(\boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t \right)^2} \right\} \right] dt \\ &+ 2\gamma \frac{\boldsymbol{z}_t^\top \boldsymbol{Q} \mathrm{diag}(\boldsymbol{z}_t)}{\boldsymbol{z}_t^\top \boldsymbol{Q} \boldsymbol{z}_t} d\boldsymbol{W}_t \end{aligned}$$