Production Networks and R&D Allocation

Yasutaka Koike
UNC Chapel Hill
TDB-CAREE

Koki Okumura
CUHK
TDB-CAREE

Hong Kong Junior Macro Group

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How do production networks shape R&D misallocation?

- New products can inherit the firm's existing supplier and buyer relationships.
- For example, TSMC can manufacture a new chip using equipment from an existing supplier such as ASML and supply it to an existing customer such as NVIDIA.
- Therefore, production networks can be a key determinant of returns to R&D.
- We analyze how production networks shape the private and social returns to R&D.

Main findings

Theoretical results

The following R&D reallocations raise welfare:

- From entrants to incumbents.
- From younger to older incumbents with more extensive trading networks.

Sources of R&D misallocation

- **Surplus appropriation.** Innovators capture only part of the surplus their new products generate for buyers.
- **Matching congestion.** Creating new products reduces other firms' opportunities to form trading relationships.

Quantification

We estimate the model using Japanese firm-to-firm transaction and patent data.

- The planner reduces stationary entry by 54%.
- The transition yields a 2.8% consumption-equivalent welfare gain.

Related literature

- **Production networks** shape aggregate fluctuations and allocative efficiency.
  Acemoglu et al. (2012); Baqaee and Farhi (2020).

- **Innovation** drives product creation, firm dynamics, and growth.
  Romer (1990); Klette and Kortum (2004).

We study how production networks shape R&D allocation.

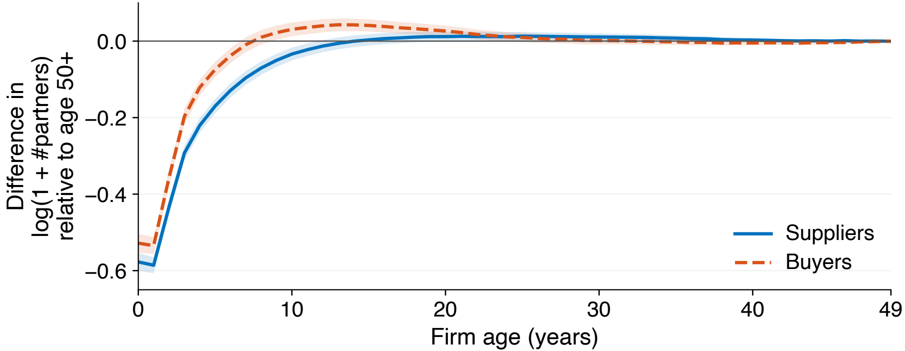
Related literature

- **Customer capital** and investment in demand
  Gourio and Rudanko (2014); Ignaszak and Sedláček (2026).
  New products inherit both supplier and buyer relationships.

- **Production network dynamics** over the firm life cycle
  Aekka and Khanna (2025); Asai and Nirei (2026).
  Accumulated supplier and buyer relationships shape R&D allocation.

Trading networks expand early in the firm life cycle

- Japanese manufacturing firms in TDB, 1998–2021.
- From age 1 to 10, supplier and buyer networks expand by approximately 75%.



9,894,357 firm-year observations. Outcome: $\log(1 + \text{partners})$. Firm and industry×year FE; controls: logs of employment, capital and sales per worker. Shading: pointwise 95% intervals using firm-clustered SEs.

Model

Model environment

- Continuous time. Household utility over consumption $Y(t)$ is $\int_0^\infty e^{-\rho t} \log Y(t)\, dt$.
- One unit of production labor and one unit of R&D labor.
- Firms own multiple differentiated product lines (Klette and Kortum, 2004).
- Production is roundabout. Each product line uses labor and inputs from linked suppliers and supplies both final consumers and linked buyers.
- Supplier links expand input varieties and lower production costs. Buyer links generate intermediate demand.

New products inherit their parent product line's network

- An incumbent with n product lines chooses a per-product-line innovation rate λ.

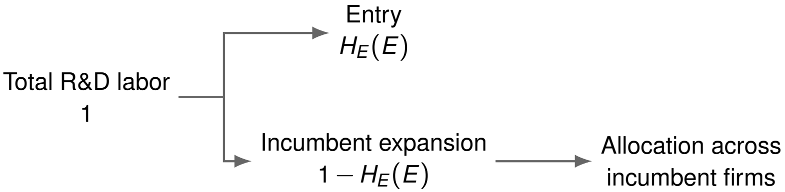
$$\text{New-product creation rate} = n\lambda, \qquad \text{R\&D labor} = nh_H(\lambda), \qquad h_H(\lambda) = \frac{\lambda^\gamma}{\varphi}.$$

- Each new product line inherits its parent line's supplier and buyer relationships.

Entry and R&D allocation

- An entry flow E requires $H_E(E)$ units of R&D labor and creates single-product firms with initial link intensity ζ_0.

$$H_E(E) = \frac{E^{\gamma_E}}{\varphi_E}$$



Network dynamics

- $f(a)$ is the density of product lines owned by firms of age a.
- Each product line meets new suppliers at a mass flow rate of ζ. Suppliers are drawn according to the normalized product-line density $f(a)/N$, where $N = \int_0^\infty f(a)\,da$.
- Firms exit at rate δ_F and product lines at rate δ_P, removing their attached links. Individual links dissolve at rate δ_M.
- Tracking trading partners, their partners, and so on makes the state space explode.
- Product lines owned by firms of the same age face the same partner-age distributions.
- Firm size remains stochastic.

Stationary laws of motion

- Incumbent innovation and exits shape the distribution of product lines across firm ages.

$$f'(a) = [\lambda(a) - d]f(a), \quad f(0) = E, \quad d = \delta_F + \delta_P.$$

- $m(a_s, a_b)$ is the supplier-line density per buyer line, where a_s and a_b are the ages of the supplier and buyer firms.

$$(\partial_{a_s} + \partial_{a_b})m = \underbrace{\lambda(a_s)m}_{\substack{\text{Supplier R\&D} \\ \text{and inheritance}}} - \underbrace{(d + \delta_M)m}_{\substack{\text{Supplier exits} \\ \text{and link dissolution}}} + \underbrace{\zeta \frac{f(a_s)}{N}}_{\text{New meetings}}.$$

- Entrants start with initial link intensity ζ_0.

$$m(a_s, 0) = \zeta_0 \frac{f(a_s)}{N}, \qquad m(0, a_b) = \zeta_0 \frac{E}{N}.$$

Stationary distributions

- The density of product lines owned by firms of age a.

$$f(a) = E \cdot \exp\left\{ \int_0^a [\lambda(s) - d]\, ds \right\}.$$

- The density of supplier product lines owned by firms of age a_s, per product line owned by a firm of age a_b.

$$m(a_s, a_b) = \underbrace{\left[\frac{\zeta}{\delta_M} - \left(\frac{\zeta}{\delta_M} - \zeta_0 \right) e^{-\delta_M \min\{a_s, a_b\}} \right]}_{\text{Age-dependent connectivity}} \underbrace{\frac{f(a_s)}{N}}_{\substack{\text{Supplier-age}\\ \text{distribution}}} .$$

- As the younger firm's age, $\min\{a_s, a_b\}$, increases, connectivity converges from ζ_0 to ζ/δ_M.

Production and contracting

- A Cobb–Douglas upper nest combines labor and a CES bundle of linked suppliers' inputs. The labor exponent is β, and the substitution elasticity is σ.
- **Love of variety:** Access to an additional input lowers the buyer's cost
- **Two-part tariffs:** Buyers pay marginal cost for each unit of input and a fixed transfer to the supplier. The transfer gives the supplier half of the bilateral surplus $(\theta = 1/2)$.
- Firms charge the final-good producer a common markup of $\sigma/(\sigma-1)$.
- The decentralized goods allocation coincides with the planner's for a given network.

Private innovation incentives

Firm value is $nV(a)$, where $V(a)$ is the private value per product line.

$$(\rho + d)V(a) = \underbrace{\pi(a)}_{\text{Current payoff}} + \underbrace{V_a(a)}_{\text{Aging}} + \underbrace{\max_{\lambda \geq 0}\{\lambda V(a) - w_H h_H(\lambda)\}}_{\text{Innovation}}.$$

$\pi(a)$ includes operating profits and net fixed transfers; w_H is the R&D wage.

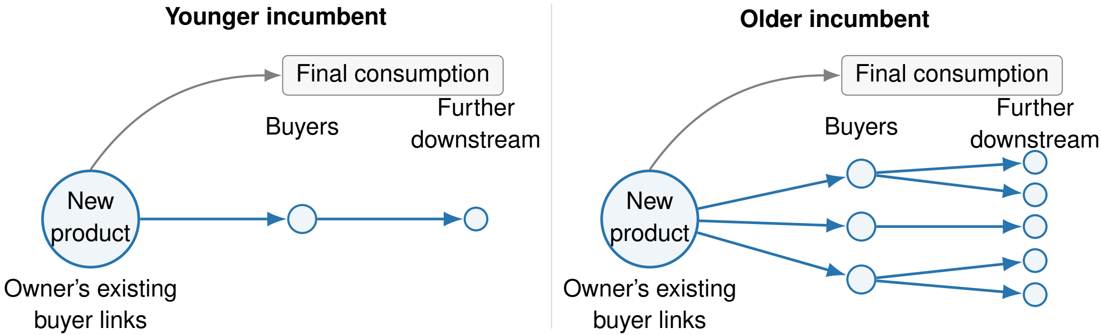
At an interior decentralized equilibrium,

$$\underbrace{\frac{V(a)}{h_H'(\lambda(a))}}_{\text{Incumbent expansion at age } a} = \underbrace{\frac{V(0)}{H_E'(E)}}_{\text{Entry}} = w_H.$$

Marginal private returns per R&D worker

R&D Misallocation

Inherited buyer links extend the reach of new products



- **Limited appropriability:** Innovators capture only part of the surplus their new products generate for buyers.

Private returns underweight established buyer relationships

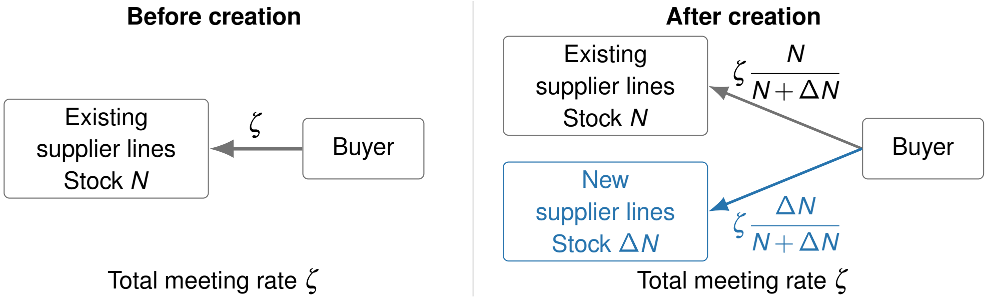
- Older firms' new products serve more intermediate buyers.
- Innovators capture only part of the benefits generated for these buyers.
- As firms age and accumulate buyer relationships, private instantaneous returns capture a smaller share of their social counterparts.

$$\frac{\pi(a)}{\pi^{SP}(a)} = \frac{\sigma-1}{\sigma}\left[\theta\beta + (1-\theta)\underbrace{\frac{1}{D(a)}}_{\substack{\text{Relative importance of}\\\text{final-consumption sales}}}\right].$$

$\pi^{SP}(a)$ is the instantaneous social return.
θ is the share of bilateral surplus captured by the supplier.

Congestion costs are larger relative to returns at entry

- New product lines divert meeting opportunities from existing supplier lines.



- Entry and incumbent expansion impose the same current cost per added line, Ω/N.
- Inherited relationships increase the social current return per product line, $\pi^{SP}(a)$.

Private and social product-line values

Decentralized equilibrium

$$(\rho + d - \lambda(a))V(a) = \pi(a) + V_a(a) - w_H h_H(\lambda(a)).$$

Stationary planner

$$(\rho + d - \lambda^{SP}(a))V^{SP}(a) = \pi^{SP}(a) + V_a^{SP}(a) - w_H^{SP} h_H(\lambda^{SP}(a)) - \frac{\Omega}{N}.$$

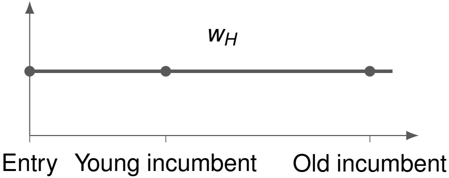
w_H^{SP} is the shadow wage of R&D labor.

- **Appropriability:** Private payoffs capture only part of the social return.
- **Matching congestion:** The planner accounts for lost matching opportunities elsewhere.

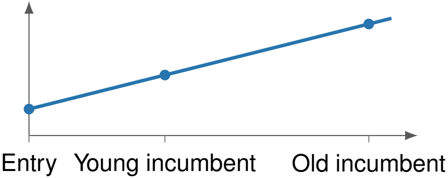
Two welfare-improving R&D reallocations

Both returns are evaluated at the stationary decentralized equilibrium.

Private marginal return per R&D worker



Social marginal return per R&D worker



Marginal R&D reallocations raise welfare:

- Entrants $\longrightarrow$ incumbents.
- Younger $\longrightarrow$ older incumbents, holding entry R&D labor fixed.

Quantification

Patenting is positively related across trading partners

$$\log P_{it} = \beta_S \log\left(\frac{P_{it}^S}{S_{i,t-1}}\right) + \beta_B \log\left(\frac{P_{it}^B}{B_{i,t-1}}\right) + \alpha_i + \delta_t + \varepsilon_{it}$$

P_{it} counts firm i's patent applications in year t. P_{it}^S and P_{it}^B sum year-t patent applications by suppliers and buyers observed at $t-1$; $S_{i,t-1}$ and $B_{i,t-1}$ count those partners.

	OLS	IV
Supplier patenting per supplier	0.017 (0.003)	0.074 (0.026)
Buyer patenting per buyer	0.019 (0.003)	0.173 (0.041)

- IV uses partners' lagged patent-class exposure to foreign patenting.
- In the model, supplier innovation expands input access and buyer innovation expands demand.

TDB–IIP, 1996–2019. 50,918 firm-years with positive own and partner patenting. Firm and year FE; industry × year clustered SE in parentheses. Joint-IV first-stage F = 55.3.

IV construction Full results

Estimation targets

Moment	Main parameters informed
Average links per firm	Matching efficiency, ζ
Age profile of supplier and buyer links	Initial link intensity, ζ_0
Average partner-patenting IV coefficient	Incumbent R&D efficiency, φ

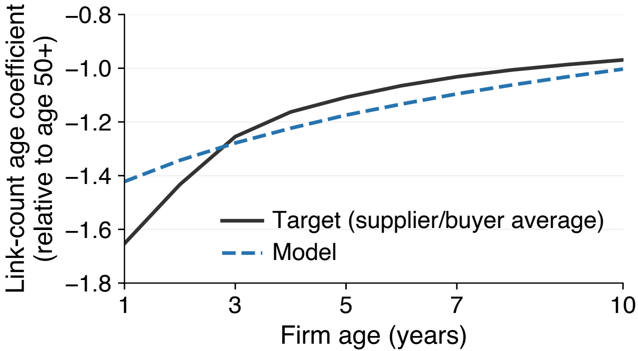
Links per firm: Imani and Ohyama (2022). Age coefficients based on cell mean link counts: TDB, 1998–2019.

Calibrated and estimated parameters

Parameter	Value	Notes
Discount rate ρ	0.05	
CES elasticity σ	3	Markup on final sales: 1.5
Primary-factor share of variable costs β	0.33	Japanese input-output data
Supplier share of bilateral surplus θ	0.5	Equal surplus sharing
Firm exit rate δ_F	0.04	Japanese firm exit data
Product-line exit rate δ_P	0.06	Product-dropping probability
Link dissolution rate δ_M	0.073	Link-stock survival
R&D cost curvatures $\gamma = \gamma_E$	2	Quadratic costs
Matching rate ζ	2.3	Estimated
Link intensity at entry ζ_0	12	Estimated
Incumbent R&D efficiency φ	0.18	
Entry R&D efficiency φ_E	3.9	

Time is measured in years. R&D efficiencies are reported in units such that decentralized steady-state entry equals one.

Estimates and targeted fit

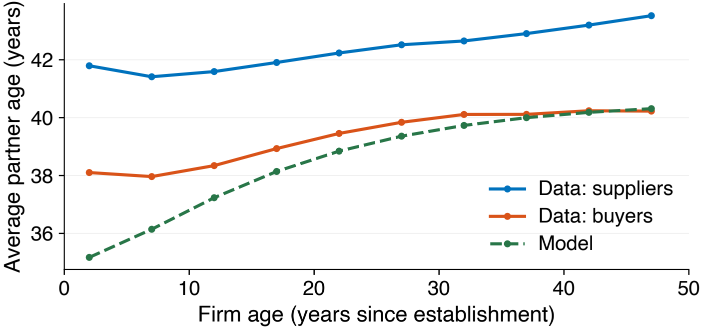


Moment	Data	Model
Buyer-supplier links per firm	27.3	27.3
Average partner-patenting coefficient	0.124	0.118

Data: age coefficients controlling for year, prefecture, and industry effects.
Model: implied age profile. Both relative to age 50+.

- In the estimated model, initial link intensity is 36% of the long-run benchmark.

Untargeted partner-age profiles



- The model captures broad partner-age levels and their upward age pattern.

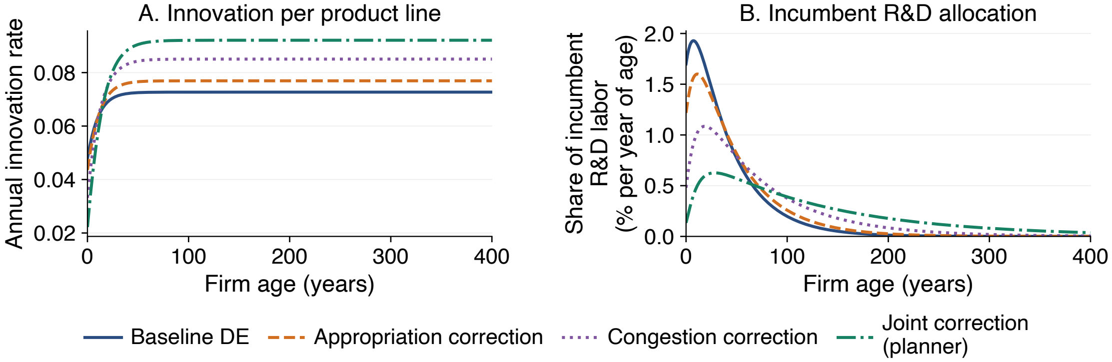
TDB, 1998–2019, across industries. Establishment ages; five-year groups. Joint age, both endpoints' prefecture/industry, and year effects; equal relationship weights.

The planner shifts R&D from entry to incumbents

Stationary allocation	Decentralized	Planner
Entry R&D share (%)	26.2	5.7
Entry flow (DE = 1)	1.00	0.46
Innovation rate per existing product line (% per year)	6.7	8.1

R&D allocation under separate and joint corrections

- **Appropriation correction** gives suppliers the full bilateral surplus.
- **Congestion correction** taxes each product line to internalize congestion.
- Both corrections shift R&D toward older incumbents.



Welfare gains from R&D reallocation

- The planner transition yields a **2.8%** consumption-equivalent welfare gain.
- Reallocation between entrants and incumbents accounts for **66%** of the allocation gain.
- Reallocation across incumbent ages accounts for the remaining **34%**.

A uniform incumbent R&D subsidy captures 83% of the planner gain

- We subsidize R&D for product-line expansion after entry, financed by a lump-sum tax.
- The optimal uniform subsidy of **54%** yields a **2.3%** consumption-equivalent gain.
- The uniform subsidy shifts R&D from entrants to incumbents.
- Greater incumbent expansion allows firms to accumulate more product lines as they age.
- This shifts incumbent R&D toward older firms.
- An entry tax implements the same allocation.

Conclusion

- Incumbents' new products inherit accumulated supplier and buyer relationships.
- Innovators capture only part of the benefits to buyers and do not internalize matching congestion.
- R&D reallocations from entrants to incumbents and from younger to older incumbents raise welfare.
- The planner transition yields a 2.8% welfare gain. Of the allocation gain, $2/3$ comes from entrants versus incumbents and $1/3$ from reallocation across incumbents.

Appendix

Appendix contents

- Additional evidence
- Model details
- R&D misallocation
- Quantitative details

Foreign-patenting instruments

$$Z_{it}^{S} = \sum_{c} \frac{P_{ic,t-1}^{S}}{P_{i,t-1}^{S}} P_{ct}^{\text{for}}, \qquad Z_{it}^{B} = \sum_{c} \frac{P_{ic,t-1}^{B}}{P_{i,t-1}^{B}} P_{ct}^{\text{for}}.$$

- c indexes patent classes. $P_{ic,t-1}^{S}/P_{i,t-1}^{S}$ is suppliers' lagged patent share in class c; the buyer share is analogous.
- P_{ct}^{for} counts foreign triadic patent families in class c with first JPO application in year t.
- Robustness checks exclude patents citing partners and control for own foreign-patenting exposure.

Sources: OECD Triadic Patent Families, IIP and matched TDB networks.

Back to partner patenting

Full partner-patenting results

	OLS			IV		
	(1)	(2)	(3)	(4)	(5)	(6)
Supplier patenting per supplier	0.019***		0.017***	0.083***		0.074***
	(0.003)		(0.003)	(0.025)		(0.026)
Buyer patenting per buyer		0.021***	0.019***		0.175***	0.173***
		(0.003)	(0.003)		(0.040)	(0.041)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
First-stage F				196.8	108.0	55.3
Observations	50,918	50,918	50,918	50,918	50,918	50,918

Dependent variable: log patent applications. Common positive-patenting IV sample. Industry $\times$ year clustered SE in parentheses. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Back to partner patenting

Production technology

Product lines are indexed by $\omega \in \mathscr{P}$; $x(\omega', \omega)$ is the intermediate input supplied by ω' to ω.

$$x(\omega) = \frac{l(\omega)^{\beta}}{\beta^{\beta}(1-\beta)^{1-\beta}} \left(\int_{\omega' \in \mathscr{S}(\omega)} x(\omega', \omega)^{\frac{\sigma-1}{\sigma}} d\omega' \right)^{\frac{\sigma(1-\beta)}{\sigma-1}}.$$

$$Y = \left(\int_{\omega \in \mathscr{P}} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \qquad x(\omega) = y(\omega) + \int_{\omega' \in \mathscr{B}(\omega)} x(\omega, \omega') d\omega'.$$

- $x(\omega)$ is gross output, $y(\omega)$ final-consumption use, and $l(\omega)$ production labor. β is the labor exponent and $\sigma > 1$ the substitution elasticity.
- $\mathscr{S}(\omega)$ and $\mathscr{B}(\omega)$ are supplier and buyer product-line sets. Supplier access expands the input bundle; buyer access raises intermediate demand.

Network state and matching

Product lines owned by firms of the same age face the same partner-age distributions. a_s and a_b are supplier and buyer firm ages. $f(a)$ is product-line density by owner age; $m(a_s, a_b)$ is supplier-line density per buyer line.

$$N = \int_0^\infty f(a)\,da, \qquad d = \delta_F + \delta_P, \qquad f'(a) = [\lambda(a) - d]f(a), \quad f(0) = E.$$

$$(\partial_{a_s} + \partial_{a_b})m(a_s, a_b) = [\lambda(a_s) - d - \delta_M]m(a_s, a_b) + \zeta\frac{f(a_s)}{N}.$$

$$m(a_s, 0) = \zeta_0\frac{f(a_s)}{N}, \qquad m(0, a_b) = \zeta_0\frac{E}{N}.$$

- $\lambda(a)$ is innovation per line; E is entry. Firm and product-line exit rates are δ_F and δ_P.
- Each line meets suppliers at rate ζ. Links dissolve at rate δ_M; entrants have initial link intensity ζ_0.
- The equations describe a stationary network. Innovation reproduces inherited relationships.

Deriving the stationary matching distribution

- Dividing by supplier-line density cancels supplier innovation and exit terms.

$$k(a_s, a_b) \equiv \frac{m(a_s, a_b)}{f(a_s)}, \qquad (\partial_{a_s} + \partial_{a_b})k = -\delta_M k + \frac{\zeta}{N}.$$

- Both ages increase together. Tracing backward reaches an entry boundary after $\min\{a_s, a_b\}$ years.

$$k(a_s, 0) = k(0, a_b) = \frac{\zeta_0}{N}.$$

- Integrating along this characteristic gives

$$k(a_s, a_b) = \frac{1}{N}\left[\frac{\zeta}{\delta_M} + \left(\zeta_0 - \frac{\zeta}{\delta_M}\right) e^{-\delta_M \min\{a_s, a_b\}}\right].$$

- Recover $m(a_s, a_b) = f(a_s)k(a_s, a_b)$. Symmetry of k implies reciprocity:

$$m(a_s, a_b)f(a_b) = m(a_b, a_s)f(a_s).$$

Back to stationary distributions

Prices and input expenditure weights

At a given network (f, m), $c(a)$ is unit production cost and w is the production wage. Intermediate inputs are priced at marginal cost $c(a)$. Final sales carry a common markup. Contracts are end-use contingent, and resale across uses is ruled out.

$$p^F(a) = \frac{\sigma}{\sigma - 1} c(a).$$

$$c(a_b) = w^{\beta} \left[\int c(a_s)^{1-\sigma} m(a_s, a_b)\, da_s \right]^{\frac{1-\beta}{1-\sigma}}.$$

$$s^M(a_s, a_b) = \frac{c(a_s)^{1-\sigma}}{\int c(\tilde{a})^{1-\sigma} m(\tilde{a}, a_b)\, d\tilde{a}}, \qquad \int s^M(a_s, a_b) m(a_s, a_b)\, da_s = 1.$$

- s^M is the input-expenditure weight per supplier variety.
- The share spent on suppliers in an age interval is $s^M(a_s, a_b) m(a_s, a_b)\, da_s$.

Sales and demand

Normalize final expenditure to $P^F Y = 1$. Final-sales revenue per product line is

$$P^F = \left[\int p^F(a)^{1-\sigma} f(a)\, da\right]^{\frac{1}{1-\sigma}}, \qquad R^F(a) = \frac{p^F(a)y(a)}{P^F Y} = \left(\frac{p^F(a)}{P^F}\right)^{1-\sigma}.$$

The demand index $D(a) \equiv x(a)/y(a)$ is gross output relative to final demand. It satisfies

$$D(a) = \underbrace{1}_{\text{Final demand}} + \underbrace{(1-\beta)\int \frac{R^F(a_b)}{R^F(a)} s^M(a, a_b) D(a_b) m(a_b, a)\, da_b}_{\text{Demand through buyers}}.$$

Reciprocity gives buyer exposure per supplier line as $m(a_b, a)$. Variable production cost is

$$C(a) \equiv \frac{c(a)x(a)}{P^F Y} = \frac{\sigma - 1}{\sigma} R^F(a) D(a).$$

With one unit of production labor,

$$w = \beta \int C(a) f(a)\, da = \frac{\sigma - 1}{\sigma}.$$

Bilateral surplus and transfers

Expenditure on one supplier variety is $(1-\beta)C(a_b)s^M(a_s,a_b)$.

$$S(a_s,a_b) = \frac{1-\beta}{\sigma-1}C(a_b)s^M(a_s,a_b), \qquad T(a_s,a_b) = \theta S(a_s,a_b), \quad \theta = \frac{1}{2}.$$

- This input-variety benefit creates bilateral surplus S: the buyer's marginal variable-cost saving from an additional supplier variety, holding output and input prices fixed.
- Under CES demand, this saving equals expenditure on the variety divided by $\sigma-1$.
- Intermediate sales cover the supplier's variable cost. The supplier captures share θ of the surplus through the fixed transfer T.
- Fixed transfers split surplus without changing input choices.

Private flow payoff

The current payoff per product line combines final-sales profit and net fixed transfers.

$$\pi(a) = \underbrace{\frac{R^F(a)}{\sigma}}_{\text{Final-sales profit}} + \underbrace{\theta \int S(a, a_b) m(a_b, a)\, da_b}_{\text{Buyer transfers}} - \underbrace{\theta \int S(a_s, a) m(a_s, a)\, da_s}_{\text{Supplier transfers}}.$$

Using goods-market clearing and $\int s^M(a_s, a) m(a_s, a)\, da_s = 1$,

$$\underbrace{\theta \int S(a, a_b) m(a_b, a)\, da_b}_{\text{Receipts}} = \frac{\theta}{\sigma - 1}\left[C(a) - \frac{\sigma - 1}{\sigma} R^F(a)\right], \quad \underbrace{\theta \int S(a_s, a) m(a_s, a)\, da_s}_{\text{Payments}} = \frac{\theta(1-\beta)}{\sigma - 1} C(a).$$

Substituting $C(a) = (\sigma - 1) R^F(a) D(a)/\sigma$ gives

$$\pi(a) = \frac{1-\theta}{\sigma} R^F(a) + \frac{\theta\beta}{\sigma - 1} C(a) = \frac{R^F(a)}{\sigma}\left[(1-\theta) + \theta\beta D(a)\right].$$

Buyer demand enters private returns through the supplier's surplus share θ.

Firm dynamics and private value

An age-a firm owns $n \geq 1$ lines. $\pi(a)$ is payoff per line, w_H the R&D wage, and $h_H(\lambda) = \lambda^\gamma/\varphi$. Here ρ is the discount rate, $\gamma > 1$ R&D cost curvature, and φ R&D efficiency. With $V^F(0,a) = 0$,

$$\begin{aligned}(\rho + \delta_F)V^F(n,a) = {} & n\pi(a) + \partial_a V^F(n,a) \\ & + n\delta_P\left[V^F(n-1,a) - V^F(n,a)\right] \\ & + \max_{\lambda \geq 0}\left\{n\lambda\left[V^F(n+1,a) - V^F(n,a)\right] - nw_H h_H(\lambda)\right\}.\end{aligned}$$

Lines share age-specific payoff opportunities. Payoffs, R&D costs, and line transition rates scale with n. Substitute $V^F(n,a) = nV(a)$ and divide by n:

$$(\rho + d)V(a) = \pi(a) + V_a(a) + \max_{\lambda \geq 0}\{\lambda V(a) - w_H h_H(\lambda)\}, \qquad d = \delta_F + \delta_P.$$

- A new product line has value $V(a)$ when created by an age-a incumbent, and $V(0)$ when created by an entrant.

Computing the stationary decentralized equilibrium

Fix parameters and an age grid. Guess $(\lambda(a), E, w_H)$; normalize $P^F Y = 1$.

1. **Stationary network.** Use the closed forms to compute product-line density f, total mass N, and supplier density m from (λ, E).
2. **Costs and profits.** At $w = (\sigma - 1)/\sigma$, solve for unit costs c, then prices, input weights and final sales R^F. Solve the linear cost recursion for C and compute private profits π.
3. **Firm value.** Given w_H and π, solve for V and optimal innovation on the age grid.
4. **R&D and entry.** Entry uses $H_E(E) = E^{\gamma_E}/\varphi_E$ units of R&D labor, with curvature $\gamma_E > 1$ and efficiency φ_E. Given V and f, update w_H to clear total R&D labor supply of one:

$$\lambda(a) = \left[\frac{\varphi V(a)}{\gamma w_H}\right]^{\frac{1}{\gamma-1}}, \qquad E = \left[\frac{\varphi_E V(0)}{\gamma_E w_H}\right]^{\frac{1}{\gamma_E-1}},$$

$$\frac{E^{\gamma_E}}{\varphi_E} + \int \frac{\lambda(a)^{\gamma}}{\varphi} f(a)\, da = 1.$$

Damp the updates to (λ, E, w_H) and repeat until convergence; check equilibrium residuals.

Back to private innovation incentives Firm dynamics Stationary distributions

Planner's problem

$$\max_{\{E_t,\lambda_t(\cdot)\}_{t\geq 0}} \int_0^\infty e^{-\rho t} \log Y(f_t, m_t)\, dt.$$

$Y(f_t, m_t)$ is maximum final consumption at the given network state, subject to production technology, goods-market clearing, and the fixed production-labor resource.

$$H_E(E_t) + \int_0^\infty h_H(\lambda_t(a)) f_t(a)\, da = 1, \qquad E_t \geq 0, \quad \lambda_t(a) \geq 0.$$

$$h_H(\lambda) = \frac{\lambda^\gamma}{\varphi}, \qquad H_E(E) = \frac{E^{\gamma_E}}{\varphi_E}.$$

- Product-line and link densities follow their laws of motion and entry boundary conditions, starting from a given initial state.

Network structure and social current returns

At a given network state (f, m), with the production wage normalized to one,

$$c^{SP}(a_b) = \left[\int c^{SP}(a_s)^{1-\sigma} m(a_s, a_b)\, da_s\right]^{\frac{1-\beta}{1-\sigma}}.$$

The demand index $D^{SP}(a) \equiv x^{SP}(a)/y^{SP}(a)$ measures gross output relative to final demand:

$$D^{SP}(a_s) = 1 + (1-\beta) \int c^{SP}(a_b)^{\frac{\beta(\sigma-1)}{1-\beta}} D^{SP}(a_b) m(a_b, a_s)\, da_b.$$

$$P^{SP} = \left[\int c^{SP}(a)^{1-\sigma} f(a)\, da\right]^{\frac{1}{1-\sigma}}, \qquad \pi^{SP}(a) = \frac{1}{\sigma-1}\left(\frac{c^{SP}(a)}{P^{SP}}\right)^{1-\sigma} D^{SP}(a).$$

- Supplier access determines unit cost c^{SP}.
- The constant one is final demand; the integral adds demand through buyers and their downstream relationships.

Back to current-return ratio

Private and social current returns

At a common network (f,m), both allocations share the demand index $D(a) = D^{SP}(a)$, and

$$R^F(a) = \left(\frac{c^{SP}(a)}{P^{SP}}\right)^{1-\sigma}.$$

The same demand index enters social and private current returns:

$$\pi^{SP}(a) = \frac{R^F(a)D(a)}{\sigma - 1},$$

$$\pi(a) = \frac{R^F(a)}{\sigma}\left[(1-\theta) + \theta\beta D(a)\right].$$

$$\frac{\pi(a)}{\pi^{SP}(a)} = \frac{\sigma-1}{\sigma}\left[\theta\beta + \frac{1-\theta}{D(a)}\right].$$

- $1/D(a)$ is the share of gross output used for final consumption.
- For $\theta < 1$, a higher demand index lowers the return ratio. At $\theta = 1$, it equals $\beta(\sigma-1)/\sigma$ at every age.

Back to current-return ratio Private flow payoff

Social match values

$$
\begin{aligned}
&[\rho + 2d + \delta_M - \lambda^{SP}(a_s) - \lambda^{SP}(a_b)] V^M(a_s, a_b) \\
&\quad = (\partial_{a_s} + \partial_{a_b}) V^M(a_s, a_b) + (1-\beta)\pi^{SP}(a_b) s(a_s, a_b).
\end{aligned}
$$

$$
s(a_s, a_b) = \frac{c^{SP}(a_s)^{1-\sigma}}{\int c^{SP}(\tilde{a})^{1-\sigma} m(\tilde{a}, a_b) d\tilde{a}}.
$$

- V^M values an additional supplier variety in a buyer's input menu.
- s is the input-expenditure weight per supplier variety. The share for a supplier-age interval is $s(a_s, a_b) m(a_s, a_b)\, da_s$.
- At the same network state, baseline marginal pricing gives $s = s^M$. The flow payoff is the input-variety benefit net of the resource cost of supplying the input.

Back to product-line values

The cost of matching congestion

$$\Omega = \underbrace{\frac{\zeta}{N}\iint V^M(a_s, a_b) f(a_s) f(a_b) da_s da_b}_{\text{Matches between incumbents}}$$
$$+ \underbrace{\frac{\zeta_0 E}{N}\int V^M(a_s, 0) f(a_s) da_s}_{\text{Entrants as buyers}}$$
$$+ \underbrace{\frac{\zeta_0 E}{N}\int V^M(0, a_b) f(a_b) da_b}_{\text{Entrants as suppliers}}.$$

- Each term weights new matching flows by their social value.
- Adding a product line imposes a matching congestion cost of Ω/N.

Back to matching congestion

Age-independent networks eliminate R&D misallocation

Consider the case

$$\zeta_0 = \frac{\zeta}{\delta_M}.$$

- Supplier and buyer exposure per product line are the same at every firm age.
- The stationary decentralized and planner R&D allocations coincide.
- Private–social return gaps remain, but do not distort relative innovation incentives.

Back to R&D reallocations

Targeted fit details

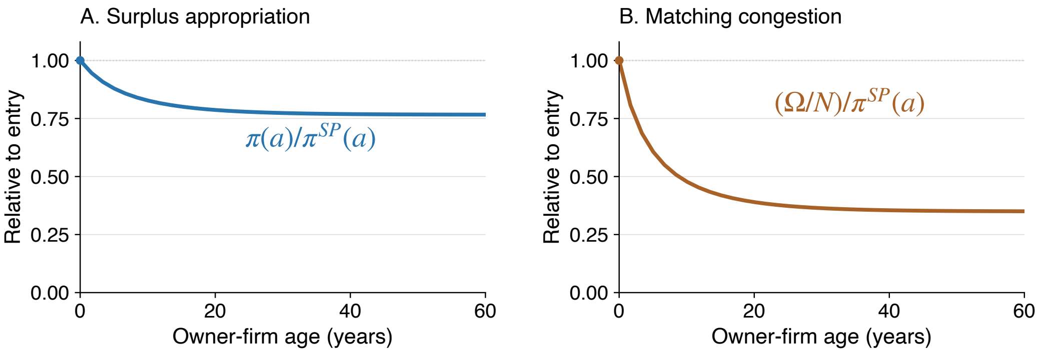
Moment	Data	Model
Buyer-supplier links per firm	27.3	27.3
Average partner-patenting coefficient	0.124	0.118
Age-profile RMSE (log points)		0.092

- The life-cycle block averages the supplier and buyer log-count coefficients at each age, giving ten targets for ages 1–10 relative to 50+.
- The empirical target averages the supplier and buyer joint-IV patenting coefficients. The model counterpart averages their joint log–log OLS coefficients using innovation intensities per product line.
- Reported model coefficients average across ten simulation seeds unused in estimation.

Back to estimates and fit

Current-return ratios by firm age

Both ratios are evaluated at the estimated stationary decentralized equilibrium.



Back to matching congestion

Separate and joint corrections

	DE	Appropriation	Congestion	Planner
Entry R&D share (%)	26.2	21.8	12.6	5.7
Mean incumbent age (years, R&D weighted)	39.7	47.6	73.7	134.9

- Appropriation correction gives suppliers the full bilateral surplus.
- Congestion correction imposes a tax equal to the matching congestion cost, in private-value units, on active product lines.
- Joint correction lowers the entry R&D share by 2.6 percentage points more than the sum of separate corrections.

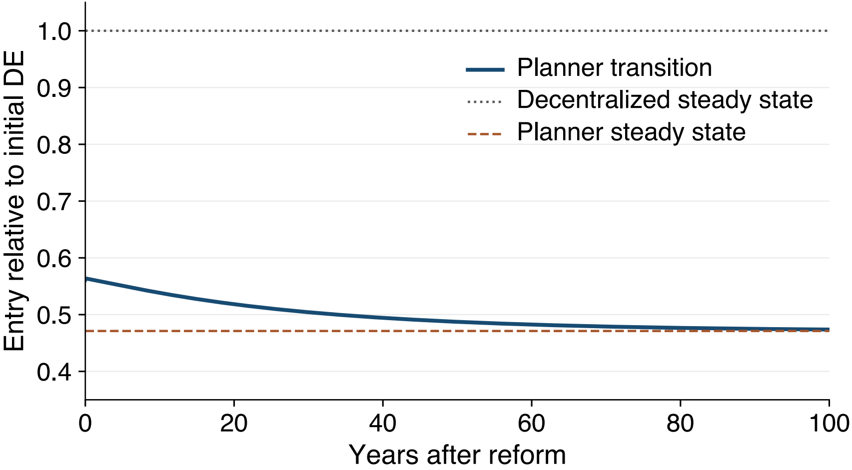
Congestion correction amplifies the shift toward older incumbents

Effect of correcting appropriation on the R&D-weighted mean incumbent age, comparing stationary allocations.

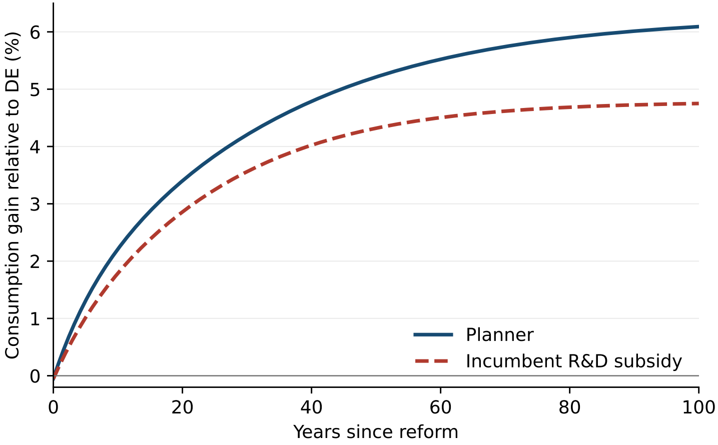
Congestion correction	Increase in mean age
Not applied	+7.9 years
Applied	+61.2 years

Back to allocation comparisons

Entry along the planner transition



Consumption gains along the transition



Back to welfare gains

Consumption-equivalent welfare

Let U_{path} be discounted log consumption along a reform path and U_{DE} the stationary decentralized benchmark.

$$\Delta_{\mathrm{CE}} = \exp\{\rho(U_{\text{path}} - U_{DE})\} - 1.$$

- Δ_{CE} is the permanent proportional increase in baseline consumption that gives the same welfare.
- The subsidy captures 83% of the planner gain, comparing $\log(1 + \Delta_{\mathrm{CE}})$.
- Shapley contributions average each margin's log-CE gain with the other margin at its decentralized and planner allocation. Shares divide these contributions by their sum.

Back to welfare gains Allocation decomposition

Welfare decomposition by R&D allocation margin

DE uses stationary decentralized labor shares; SP uses planner labor-share paths.

Case	Entry shares	Incumbent-age shares	$100\times$ log CE
L_{00}	DE	DE	0.027
L_{10}	SP	DE	1.063
L_{01}	DE	SP	0.175
L_{11}	SP	SP	2.740

$$S_E = \tfrac{1}{2}\left[(L_{10} - L_{00}) + (L_{11} - L_{01})\right],$$
$$S_A = \tfrac{1}{2}\left[(L_{01} - L_{00}) + (L_{11} - L_{10})\right].$$

- Entry accounts for 66% of $S_E + S_A$; allocation across incumbent ages accounts for 34%.

All paths start from the same DE state. L_{ij} is log CE relative to stationary DE; L_{00} reflects numerical approximation of the reference path.

Back to welfare gains Welfare definitions

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Back to related literature

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